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To cite this article: V P Neznamov 2020 *J. Phys.: Conf. Ser.* **1690** 012138

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Quantum particles and the ergosphere of the Kerr metric

V P Neznamov^{1,2}

¹FSUE "RFNC-VNIIEF", Russia, Sarov, Mira pr., 37, 607188

²National Research Nuclear University MEPhI, Moscow, Russia

E-mail: vpneznamov@mail.ru

Abstract. The existence of the ergosphere of the Kerr metric does not manifest itself in quantum equations for particles of different spins. To justify the Penrose process with energy extraction from the ergosphere, it is necessary to substantiate and prove its existence within the framework of the consistent quantum theory.

1. Introduction

The stationary Kerr metric is a GR vacuum solution [1]. The Kerr solution is characterized by a point source with mass M rotating with the angular momentum $J = Mac$. The Kerr metric has several physically significant spatial surfaces. Firstly, there are the external and internal event horizons with radii

$$(r_{\pm})_K = \frac{r_0 \pm \sqrt{r_0^2 - 4a^2}}{2}. \quad (1)$$

In (1) r_0 is the Schwarzschild horizon (the gravitational radius). Secondly, there are surfaces with radii

$$(r_{\pm})_{erg} = \frac{r_0 \pm \sqrt{r_0^2 - 4a^2 \cos^2 \theta}}{2}. \quad (2)$$

The domain between radii (2) is called the ergosphere. In this domain, the time component of a metric tensor g_{00} has a negative sign ($g_{00} < 0$). Below, we will examine the external ergosphere between radii $(r_+)_K \div (r_+)_{erg}$.

As the result of the classical analysis, Roger Penrose proposed a process of energy extraction from the ergosphere of a rotating object with the Kerr metric (the Penrose process) [2, 3]. In particular, the Penrose process allows increase in the energy of a particle entering and then leaving the ergosphere.

In this paper, we would like to emphasize the absence of extreme features of ergosphere existence with boundary surfaces $r = (r_{\pm})_{erg}$ in the quantum equations for particles of different spins. Conversely, the presence of event horizons $(r_{\pm})_K$, essentially influencing on the motion of quantum particles is obviously revealed in all the quantum equations.

Hence, it follows that for the "legality" of the classical Penrose process, it is to be obtained in the quantum theory.



This conclusion does not apply to the Lense – Thirring precession [4] and to the process frame-dragging, since these effects in GR are inherent in all rotating objects, including those not having an ergosphere.

The paper is arranged as follows. In section 2, the Kerr metric is presented with event horizons and the ergosphere, for coherence of presentation. Section 3 presents quantum-mechanical equations for spinless particles ($S = 0$), photons ($S = 1$) and fermions ($S = 1/2$) in the Kerr space-time. In section 4, we discuss the obtained results. In the Conclusions, the main conclusions of the paper are described.

In the paper, as rule, we use the system of units of $\hbar = c = 1$ and the Minkowski space-time signature

$$\eta_{\alpha\beta} = \text{diag} [1, -1, -1, -1]. \quad (3)$$

2. Kerr metric

The Kerr metric in the Boyer-Lindquist coordinates (t, r, θ, φ) [5] can be represented as

$$ds^2 = \left(1 - \frac{r_0 r}{r_K^2}\right) dt^2 + \frac{2ar_0 r}{r_K^2} \sin^2 \theta dt d\varphi - \frac{r_K^2}{\Delta_K} dr^2 - r_K^2 d\theta^2 - \left(r^2 + a^2 + \frac{a^2 r_0 r}{r_K^2} \sin^2 \theta\right) \sin^2 \theta d\varphi^2, \quad (4)$$

where $r_K^2 = r^2 + a^2 \cos^2 \theta$; $\Delta_K = r^2 f_K = r^2 \left(1 - \frac{r_0}{r} + \frac{a^2}{r^2}\right)$; $r_0 = 2GM/c^2$ is the Schwarzschild gravitational radius; c is the light velocity.

In metric (4), $g_{00} = 0$, provided that

$$r^2 - r_0 r + a^2 \cos^2 \theta = 0. \quad (5)$$

Equation (5) defines the radii of the boundary surfaces of the ergosphere $(r_{\pm})_{erg}$.

- If $r_0 > 2a$, then

$$f_K = \left(1 - \frac{(r_+)_K}{r}\right) \left(1 - \frac{(r_-)_K}{r}\right), \quad (6)$$

where $(r_{\pm})_K$ are the radii of the external and internal event horizons of the Kerr metric.

- The case of $r_0 = 2a$, $(r_+)_K = (r_-)_K = r_0/2$ corresponds to the extreme Kerr field.
- The case of $r_0 < 2a$ corresponds to the naked singularity of the Kerr field. In this case, $f_K > 0$.

3. Quantum equations for particles with different spins in the Kerr space-time

Since equation (5) for boundary surfaces of the ergosphere describes a two-dimensional surface, let us write out the quantum equations before separation of variables (r, θ) .

3.1. Klein-Gordon equation for spinless particles ($S = 0$)

$$\left[\frac{1}{\sqrt{-g}} \partial_{\sigma} (g^{\sigma\tau} \sqrt{-g} \partial_{\tau}) + \mu^2 \right] \Psi = 0, \quad (7)$$

where $g = -r_K^4 \sin^2 \theta$ is the determinant of the Kerr metric, μ is the particle mass. If $\Psi(r, \theta, \varphi, t) = \Phi(r, \theta) e^{-i\omega t} e^{im\varphi}$, then equation (7) has the form of [6]

$$\left\{ \frac{1}{\Delta_K} \left[\Delta_K a^2 \sin^2 \theta - (r^2 + a^2)^2 \right] \omega^2 - \frac{\partial}{\partial r} \left(\Delta_K \frac{\partial}{\partial r} \right) - \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{m^2}{\Delta_K \sin^2 \theta} (r^2 - r_0 r + a^2 \cos^2 \theta) - \frac{2a}{\Delta_K} \left[\Delta_K - (r^2 + a^2) \right] m\omega + r_K^2 \mu^2 \right\} \Phi(r, \theta) = 0. \quad (8)$$

3.2. Maxwell equations for photons ($S = 1$)

Let us write out the Maxwell equations in the form presented in the Chandrasekhar monograph [7]. The equations represent a system of four equations for three scalar functions. The scalar functions are related to components of the tensor of an electromagnetic field F_{ij} convoluted with the Kinnersley tetrad (l^i, n^i, m^i) [8] in the Newman-Penrose formalism [9]. Therefore,

$$\varphi_0 = F_{ij}l^i m^j; \quad \varphi_1 = \frac{1}{2}F_{ij}(l^i n^j + \bar{m}^i m^j); \quad \varphi_2 = F_{ij}\bar{m}^i n^j. \quad (9)$$

Under notations $\Phi_0 = \varphi_0$; $\Phi_1 = \varphi_1 \bar{\rho}^* \sqrt{2}$; $\Phi_2 = 2\varphi_2 (\bar{\rho}^*)^2$, where $\bar{\rho} = r + ia \cos \theta$; $\rho^* = r - ia \cos \theta$, let us present Φ_0, Φ_1, Φ_2 as

$$\begin{aligned} \Phi_0 &= \Phi_0(r, \theta) e^{-i\omega t} e^{im\varphi}, \\ \Phi_1 &= \Phi_1(r, \theta) e^{-i\omega t} e^{im\varphi}, \\ \Phi_2 &= \Phi_2(r, \theta) e^{-i\omega t} e^{im\varphi}. \end{aligned} \quad (10)$$

As the result, we will obtain the following system of equations (see [7], chapter 8, equations (13)-(16)):

$$\begin{aligned} \left(\frac{\partial}{\partial \theta} - a\omega \sin \theta + \frac{m}{\sin \theta} + \text{ctg} \theta - \frac{ia \sin \theta}{\bar{\rho}^*} \right) \Phi_0(r, \theta) &= \left(\frac{\partial}{\partial r} + \frac{iK}{\Delta_K} + \frac{1}{\bar{\rho}^*} \right) \Phi_1(r, \theta), \\ \left(\frac{\partial}{\partial \theta} - a\omega \sin \theta + \frac{m}{\sin \theta} + \frac{ia \sin \theta}{\bar{\rho}^*} \right) \Phi_1(r, \theta) &= \left(\frac{\partial}{\partial r} + \frac{iK}{\Delta_K} - \frac{1}{\bar{\rho}^*} \right) \Phi_2(r, \theta), \\ \left(\frac{\partial}{\partial \theta} + a\omega \sin \theta - \frac{m}{\sin \theta} + \text{ctg} \theta - \frac{ia \sin \theta}{\bar{\rho}^*} \right) \Phi_2(r, \theta) &= -\Delta_K \left(\frac{\partial}{\partial r} - \frac{iK}{\Delta_K} + \frac{1}{\bar{\rho}^*} \right) \Phi_1(r, \theta), \\ \left(\frac{\partial}{\partial \theta} + a\omega \sin \theta - \frac{m}{\sin \theta} + \frac{ia \sin \theta}{\bar{\rho}^*} \right) \Phi_1(r, \theta) &= -\Delta_K \left(\frac{\partial}{\partial r} - \frac{iK}{\Delta_K} + \frac{2r - r_0}{\Delta_K} - \frac{1}{\bar{\rho}^*} \right) \Phi_0(r, \theta), \end{aligned} \quad (11)$$

where $K = -(r^2 + a^2)\omega + am$.

3.3. Equations for fermions ($S = 1/2$)

First, we will examine the Dirac equation for our analysis for the components of the bispinor function obtained by Chandrasekhar (see paper [10], equations (21)-(24))

$$\begin{aligned} \left(\frac{\partial}{\partial r} + \frac{iK}{\Delta_K} + \frac{1}{\bar{\rho}^*} \right) F_1 + \frac{1}{\bar{\rho}^* \sqrt{2}} \left(\frac{\partial}{\partial \theta} - a\omega \sin \theta + \frac{m}{\sin \theta} + \frac{1}{2} \text{ctg} \theta \right) F_2 &= i\mu G_1, \\ \frac{\Delta_K}{2\rho_K^2} \left(\frac{\partial}{\partial r} - \frac{iK}{\Delta_K} + \frac{(r - r_0/2)}{\Delta_K} \right) F_2 - \frac{1}{\bar{\rho} \sqrt{2}} \left(\frac{\partial}{\partial \theta} + a\omega \sin \theta - \frac{m}{\sin \theta} + \frac{1}{2} \text{ctg} \theta + \frac{ia \sin \theta}{\bar{\rho}^*} \right) F_1 &= -i\mu G_2, \\ \left(\frac{\partial}{\partial r} + \frac{iK}{\Delta_K} + \frac{1}{\bar{\rho}} \right) G_2 - \frac{1}{\bar{\rho} \sqrt{2}} \left(\frac{\partial}{\partial \theta} + a\omega \sin \theta - \frac{m}{\sin \theta} + \frac{1}{2} \text{ctg} \theta \right) G_1 &= i\mu F_2, \\ \frac{\Delta_K}{2\rho_K^2} \left(\frac{\partial}{\partial r} - \frac{iK}{\Delta_K} + \frac{(r - r_0/2)}{\Delta_K} \right) G_1 + \frac{1}{\bar{\rho}^* \sqrt{2}} \left(\frac{\partial}{\partial \theta} - a\omega \sin \theta + \frac{m}{\sin \theta} + \frac{1}{2} \text{ctg} \theta - \frac{ia \sin \theta}{\bar{\rho}} \right) G_2 &= -i\mu F_1. \end{aligned} \quad (12)$$

In (12), μ is fermion mass. While obtaining (12), it was assumed that the wave function is $\sim e^{-i\omega t + im\varphi}$.

For discussions in section 4, we will examine the Dirac equation with the self-conjugated Hamiltonian in the Kerr field obtained in papers [11,12]. Besides, we will discuss self-conjugated equations for fermions in the Kerr field with spinor wave functions [13,14].

4. Discussions

While analyzing equations (8), (11), (12) as well as the equations from papers [11] - [14], we can emphasize the following:

- (1) In all the equations for particles with different spins, there is explicitly the following value

$$\Delta_K = r^2 f_K = (r - (r_+)_K)(r - (r_-)_K).$$

The presence of Δ_K leads to singularities on the event horizons with availability of the mode of a quantum-mechanical particle "fall" to the appropriate horizon [15].

- (2) In Maxwell equations for photons (11), in Dirac equations for fermions (12) as well as in the equations for fermions from papers [11] - [14], there are no equations (5) for boundary surfaces of the ergosphere. These equations are available only in Klein-Gordon equations (8) for scalar particles. The summand on the left side of equation (5) has the form without singularities on the ergosphere boundaries

$$\frac{m^2}{\Delta_K \sin^2 \theta} (r^2 - r_0 r + a^2 \cos^2 \theta). \quad (13)$$

Let us recall that $m = -j, -j + 1 \dots j$, where j is the quantum number of the total angular momentum. Upon attaining the external radius of the ergosphere $(r_+)_{erg}$, summand (13) becomes zero and then changes its sign. The first and the second summand of the equation (8) are also vanished at definite value of $(r, \theta, r_0, a, \omega, m, \mu)$. Obviously, in this sense, the third summand in (8) presented in expression (13) does not distinguished.

- (3) We can see that the existence of the ergosphere in the Kerr metric does not essentially manifest itself in the quantum equations for particles of different spins. Consequently, the Penrose process seems to be insufficiently substantiated. Within the framework of the existing paradigm, the existence of the Penrose process has to be initially proved and substantiated within the framework of the quantum theory. Then, upon transition to the quasi-classical and classical limits with $\hbar \rightarrow 0$, classical results of the authors of papers [2,3] have to be obtained.

The similar examination can be performed for ergospheres of other metrics with rotation, in particular, for instance, for the ergosphere of the charged Kerr-Newman metric.

5. Conclusions

The existence of the ergosphere of the Kerr metric does not noticeably manifest itself in quantum equations for particles of different spins.

To legitimize the Penrose process with extraction of energy from ergosphere, it is necessary to substantiate and prove its existence within the framework of the consistent quantum theory.

Acknowledgments

The author expresses his gratitude to A.L.Novoselova for essential technical support in preparation of the paper.

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