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Cubic-quartic optical solitons with Kudryashov's law of refractive index by *F*-expansions schemes

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ABSTRACT

This work presents cubic-quartic optical soliton solutions to Kudryashov's equation in polarization-preserving fibers. The integration is conducted with *F*-expansion scheme having four independent forms. This algorithm additionally reveals several other types of solution that includes Jacobi's elliptic functions, Weierstrass elliptic function, trigonometric function solutions as well as complexion solutions.

1. Introduction

The theory of optical solitons has made remarkable and far-reaching advances during the past few decades [1–50]. There are a wide variety of new concepts that have been introduced to bring about performance enhancement in this field with regards to telecommunications industry. These include dispersion-managed solitons, highly-dispersive optical solitons, meta-optics, ENZ materials and several others. One of the most elegant concepts that was introduced in the field of optoelectronics is "cubic-quartic (CQ) solitons" [45–50]. This idea was conceived if chromatic dispersion runs low and is consequently discarded. Therefore, the effect of dispersion, a key ingredient for the delicate balance necessary for the existence and sustaining of optical solitons, is completely eliminated. Hence, this dispersion effect is compensated with third-order dispersion (3OD) and fourth-order dispersion (4OD) terms. These led to the evolution of CQ solitons.

Later, another innovative concept that was introduced, to model the dynamics of soliton transmission through optical fibers, is a new law of refractive index by Kudryashov [1–9]. Kudryashov's equation was studied by the method of undetermined coefficients, then optical solitons were revealed and also the conservation laws were listed [1]. *F*-expansion scheme, extended trial function approach and Lie symmetry analysis yielded several types soliton solutions to the model [2,3,8]. Periodic and solitary pulses to the governing equation were secured and the effect of the nonlinearity power n on the shape of the pulse was studied [5]. Exact solutions to nonlinear nonintegrable differential equations were recovered employing Painleve approach [6]. The hierarchy of nonlinear differential equations of any order to describe the propagation pulses in a optical fiber was considered, and solitary and periodic waves were derived to the model [7]. Applying unified auxiliary equation scheme, new mapping approach and extended auxiliary equation methodology, for Kudryashov's equation, optical solitons and other solutions were acquired [9]. Today's paper combines these two novel concepts, namely CQ solitons together with Kudryashov's law of refractive index, to address the soliton dynamics in polarization-preserving fibers. Some preliminary results have already been revealed with this concept by the aid of trial function scheme [4]. Bright and singular CQ optical soliton solutions as well as solutions in terms of

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Jacobi's elliptic functions and trigonometric functions solutions are recovered by this integration scheme. Today's work studies the model with four forms of F-expansion scheme from which bright, dark and singular CQ optical solitons, dark-singular combo CQ optical solitons, singular combo CQ optical solitons and solutions in terms of Jacobi's elliptic functions, Weierstrass elliptic function, trigonometric function solutions as well as complexiton solutions emerge. It needs to be noted that these secured results are new and are being reported for the first time in this paper. These details are enumerated in subsequent sections.

1.1. Governing model

The dimensionless form of Kudryashov's equation with 3OD and 4OD reads [1–9]:

$$iq_t + iaq_{xxx} + bq_{xxxx} + \left(\frac{c_1}{|q|^{2n}} + \frac{c_2}{|q|^n} + c_3\,|q|^n + c_4\,|q|^{2n}\right)q = 0. \tag{1}$$

In (1), the complex-valued function $q(x, t)$ is the wave profile where x and t are independent spatial and temporal variables. The first term represents linear temporal evolution while the coefficients a and b are real parameters that independently controls 3OD and 4OD respectively. The next four terms, as introduced by Kudryashov, are nonlinear and stem from the law of refractive index of an optical fiber and gives self-phase modulation effect to the model. (See Table 1)

2. Mathematical analysis

To begin with, the picked structural form is:

$$q(x, t) = g(\varsigma)e^{i\phi(x,t)} \tag{2}$$

where

$$\varsigma = x - vt \tag{3}$$

and v stands for the speed of traveling waves. The phase ϕ has the split as

$$\phi = -\kappa x + \omega t + \theta \tag{4}$$

where κ is the frequency, ω is the wave number and θ is the phase constant. Next, inserting (2) into (1) yields its real part:

$$c_1 g + c_2 g^{1+n} - (\kappa^3(a - b\kappa) + \omega)g^{1+2n} + c_3 g^{1+3n} + c_4 g^{1+4n} + 3\kappa (a - 2b\kappa)g^{2n}g'' + bg^{2n}g^{(4)} = 0 \tag{5}$$

while imaginary part is

$$(v + 3a\kappa^2 - 4b\kappa^3)g' - (a - 4b\kappa)g^{(3)} = 0. \tag{6}$$

Now, differentiating (6) leads to

$$g^{(4)} = \frac{(v + 3a\kappa^2 - 4b\kappa^3)g''}{a - 4b\kappa} \tag{7}$$

and then (5) modifies to

$$c_1(a - 4b\kappa)g + c_2(a - 4b\kappa)g^{1+n} - (a - 4b\kappa)(a\kappa^3 - b\kappa^4 + \omega)g^{1+2n} + c_3(a - 4b\kappa)g^{1+3n} + c_4(a - 4b\kappa)g^{1+4n} + (3a^2\kappa + 20b^2\kappa^3 + b(v - 15a\kappa^2))g^{2n}g'' = 0. \tag{8}$$

For revealing closed form solutions, the transformation

$$g = \psi^{\frac{1}{n}} \tag{9}$$

is utilized in Eq. (8) and so

$$c_1 n^2(a - 4b\kappa) + c_2 n^2(a - 4b\kappa)\psi - n^2(a - 4b\kappa)(a\kappa^3 - b\kappa^4 + \omega)\psi^2 + c_3 n^2(a - 4b\kappa)\psi^3 + c_4 n^2(a - 4b\kappa)\psi^4 - (n-1)(3a^2\kappa + 20b^2\kappa^3 + b(v - 15a\kappa^2))(\psi')^2 + n(3a^2\kappa + 20b^2\kappa^3 + b(v - 15a\kappa^2))\psi\psi'' = 0. \tag{10}$$

3. *F*-Expansion scheme (type-I)

Assume the solution of (10) is picked as [10, 11, 13, 15]

$$\psi(\varsigma) = \sum_{j=0}^{N} \delta_j F^j(\varsigma) \tag{11}$$

where δ_j are constants which needs to be fixed and $F = F(\varsigma)$ holds

$$(F')^2 = PF^4 + QF^2 + R \tag{12}$$

where the function F is related to Jacobi elliptic functions and P, Q, R are constants. Balancing $(\psi')^2$ or $\psi\psi''$ with ψ^4 in (10) causes $N = 1$. Therefore (11) shapes up

$$\psi(\varsigma) = \delta_0 + \delta_1 F(\varsigma). \tag{13}$$

Inserting (13) into (10) yields

$$v = -\left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right),$$
$$c_1 = -\frac{(n-1)(c_3^4(n+1)^4P + 4c_3^2c_4^2\delta_1^2(n+1)^2(n+2)^2Q + 16c_4^4\delta_1^4(n+2)^4R)}{16c_4^3(n+1)(n+2)^4P},$$
$$c_2 = -\frac{c_3(n-2)(c_3^2(n+1)^2P + 2c_4^2\delta_1^2(n+2)^2Q)}{4c_4^2(n+2)^3P},$$
$$\delta_0 = -\frac{c_3(n+1)}{2c_4(n+2)}, \quad \delta_1 = \delta_1,$$
$$\omega = -\left(\kappa^3(a - b\kappa) + \frac{c_4\delta_1^2Q}{(n+1)P} + \frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right), \tag{14}$$

where

$$\alpha = 3a^2 - 15ab\kappa + 20b^2\kappa^2, \quad \beta = a - 4b\kappa. \tag{15}$$

By virtue of these recovered results, the formal solution of Kudryashov's model has the following structure:

Table 1
Comparison of Integration Algorithms.

Kudryashov's equation with 3OD and 4OD				
	F-expansion scheme (Type-I)	F-expansion scheme (Type-II)	F-expansion scheme (Type-III)	F-expansion scheme (Type-IV)
Bright soliton	Yes	No	Yes	No
Dark soliton	Yes	No	Yes	Yes
Singular soliton	Yes	Yes	Yes	Yes
Dark singular combo soliton	No	Yes	Yes	Yes
Singular combo soliton	Yes	Yes	Yes	No
Complexiton solution	Yes	Yes	Yes	Yes

$$q(x,t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} + \delta_1 F(\varsigma)\right\}^{\frac{1}{n}} \exp\left[i\left\{-\kappa x - \left(\kappa^3(a-b\kappa) + \frac{c_4\delta_1^2 Q}{(n+1)P} + \frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t + \theta\right\}\right]. \quad (16)$$

3.1. Jacobi's elliptic function solutions

Elliptic functions solutions to the adopted equation are secured by means of the solutions of (12) as below:

(1): $P = m^2, \quad Q = -(1+m^2), \quad R = 1, \quad F(\varsigma) = \mathrm{sn}\varsigma,$

$$q(x,t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} + \delta_1 \mathrm{sn}\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\kappa^3(a-b\kappa) + \frac{c_4\delta_1^2 Q}{(n+1)P} + \frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t + \theta\right\}\right]. \quad (17)$$

(2): $P = -m^2, \quad Q = 2m^2 - 1, \quad R = 1 - m^2, \quad F(\varsigma) = \mathrm{cn}\varsigma,$

$$q(x,t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} + \delta_1 \mathrm{cn}\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\kappa^3(a-b\kappa) + \frac{c_4\delta_1^2 Q}{(n+1)P} + \frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t + \theta\right\}\right]. \quad (18)$$

(3): $P = 1, \quad Q = -(1+m^2), \quad R = m^2, \quad F(\varsigma) = \mathrm{ns}\varsigma,$

$$q(x,t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} + \delta_1 \mathrm{ns}\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\kappa^3(a-b\kappa) + \frac{c_4\delta_1^2 Q}{(n+1)P} + \frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t + \theta\right\}\right]. \quad (19)$$

(4): $P = 1, \quad Q = -(1+m^2), \quad R = m^2, \quad F(\varsigma) = \mathrm{dc}\varsigma,$

$$q(x,t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} + \delta_1 \mathrm{dc}\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\kappa^3(a-b\kappa) + \frac{c_4\delta_1^2 Q}{(n+1)P} + \frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t + \theta\right\}\right]. \quad (20)$$

(5): $P = 1 - m^2, \quad Q = 2 - m^2, \quad R = 1, \quad F(\varsigma) = \mathrm{sc}\,\varsigma,$

$$q(x,t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} + \delta_1 \mathrm{sc}\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\kappa^3(a-b\kappa) + \frac{c_4\delta_1^2 Q}{(n+1)P} + \frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t + \theta\right\}\right]. \quad (21)$$

(6): $P = 1, \quad Q = 2 - m^2, \quad R = 1 - m^2, \quad F(\varsigma) = cs\varsigma,$

$$q(x,t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} + \delta_1 cs\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\kappa^3(a-b\kappa) + \frac{c_4\delta_1^2 Q}{(n+1)P} + \frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t + \theta\right\}\right]. \quad (22)$$

(7): $P = \frac{1}{4}, \quad Q = \frac{1-2m^2}{2}, \quad R = \frac{1}{4}, \quad F(\varsigma) = \mathrm{ns}\varsigma \pm cs\varsigma,$

$$q(x,t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} + \delta_1\left(\mathrm{ns}\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right] \pm cs\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\kappa^3(a-b\kappa) + \frac{c_4\delta_1^2 Q}{(n+1)P} + \frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t + \theta\right\}\right]. \quad (23)$$

(8): $P = \frac{1-m^2}{4}, \quad Q = \frac{1+m^2}{2}, \quad R = \frac{1-m^2}{4}, \quad F(\varsigma) = \mathrm{nc}\varsigma \pm \mathrm{sc}\,\varsigma,$

$$q(x,t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} + \delta_1\left(\mathrm{nc}\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right] \pm \mathrm{sc}\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\kappa^3(a-b\kappa) + \frac{c_4\delta_1^2 Q}{(n+1)P} + \frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t + \theta\right\}\right]. \quad (24)$$

(9): $P = \frac{m^2}{4}, \quad Q = \frac{m^2-2}{2}, \quad R = \frac{m^2}{4}, \quad F(\varsigma) = \mathrm{sn}\varsigma \pm i\mathrm{cn}\varsigma,$

$$q(x,t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} + \delta_1\left(\mathrm{sn}\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right] \pm i\mathrm{cn}\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\kappa^3(a-b\kappa) + \frac{c_4\delta_1^2 Q}{(n+1)P} + \frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t + \theta\right\}\right]. \quad (25)$$

(10): $P = \frac{m^2}{4}, \quad Q = \frac{m^2-2}{2}, \quad R = \frac{1}{4}, \quad F(\varsigma) = \frac{\mathrm{sn}\varsigma}{1 \pm \mathrm{dn}\varsigma},$

$$q(x,t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} + \delta_1 \frac{\mathrm{sn}\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{1 \pm \mathrm{dn}\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\kappa^3(a-b\kappa) + \frac{c_4\delta_1^2 Q}{(n+1)P} + \frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t + \theta\right\}\right]. \quad (26)$$

(11): $P = -\frac{1}{4}, \quad Q = \frac{m^2+1}{2}, \quad R = \frac{(1-m^2)^2}{4}, \quad F(\varsigma) = m\mathrm{cn}\varsigma \pm \mathrm{dn}\varsigma,$

$$q(x,t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} + \delta_1\left(m\mathrm{cn}\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right] \pm \mathrm{dn}\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\kappa^3(a-b\kappa) + \frac{c_4\delta_1^2 Q}{(n+1)P} + \frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t + \theta\right\}\right]. \quad (27)$$

(12): $P = \frac{(1-m^2)^2}{4}, \quad Q = \frac{m^2+1}{2}, \quad R = \frac{1}{4}, \quad F(\varsigma) = \mathrm{ds}\varsigma \pm cs\varsigma,$

$$q(x,t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} + \delta_1\left(\mathrm{ds}\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right] \pm cs\left[x + \left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\kappa^3(a-b\kappa) + \frac{c_4\delta_1^2 Q}{(n+1)P} + \frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t + \theta\right\}\right]. \quad (28)$$

(13):
$P > 0, \quad Q < 0, \quad R = \frac{m^2Q^2}{(1+m^2)^2P}, \quad F(\varsigma) = \sqrt{-\frac{m^2Q}{(1+m^2)P}}\,\mathrm{sn}\left(\sqrt{-\frac{Q}{1+m^2}}\,\varsigma\right),$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\sqrt{-\frac{m^2Q}{(1+m^2)P}}\,\mathrm{sn}\left(\sqrt{-\frac{Q}{1+m^2}}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{29}$$

(14):
$P<0,\quad Q>0,\quad R=\frac{(1-m^2)Q^2}{(m^2-2)^2P},\quad F(\varsigma)=\sqrt{-\frac{Q}{(2-m^2)P}}\,\mathrm{dn}\left(\sqrt{\frac{Q}{2-m^2}}\varsigma\right),$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\sqrt{-\frac{Q}{(2-m^2)P}}\,\mathrm{dn}\left(\sqrt{\frac{Q}{2-m^2}}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{30}$$

(15): $P=1,\quad Q=m^2+2,\quad R=1-2m^2+m^4,\quad F(\varsigma)=\frac{\mathrm{dn}\varsigma\mathrm{cn}\varsigma}{\mathrm{sn}\varsigma},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{31}$$

(16): $P=\frac{A^2(m-1)^2}{4},\quad Q=\frac{m^2+6m+1}{2},\quad R=\frac{(m-1)^2}{4A^2},\quad F(\varsigma)=\frac{\mathrm{dn}\varsigma\mathrm{cn}\varsigma}{A(1+\mathrm{sn}\varsigma)(1+m\mathrm{sn}\varsigma)},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{A\left(1+\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)\left(1+m\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{32}$$

(17): $P=-\frac{4}{m},\quad Q=6m-m^2-1,\quad R=-2m^3+m^4+m^2,\quad F(\varsigma)=\frac{m\mathrm{cn}\varsigma\mathrm{dn}\varsigma}{m\mathrm{sn}^2\varsigma+1},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{m\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{m\mathrm{sn}^2\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]+1}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{33}$$

(18): $P=\frac{1}{4},\quad Q=\frac{1-2m^2}{2},\quad R=\frac{1}{4},\quad F(\varsigma)=\frac{\mathrm{sn}\varsigma}{1\pm\mathrm{cn}\varsigma},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{1\pm\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{34}$$

(19): $P=\frac{1-m^2}{4},\quad Q=\frac{1+m^2}{2},\quad R=\frac{1-m^2}{4},\quad F(\varsigma)=\frac{\mathrm{cn}\varsigma}{1\pm\mathrm{sn}\varsigma},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{1\pm\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{35}$$

(20): $P=\frac{2-m^2-2m_1}{4},\quad Q=\frac{m^2-6m_1-2}{2},\quad R=\frac{2-m^2-2m_1}{4},\quad F(\varsigma)=\frac{m^2\mathrm{sn}\varsigma\mathrm{cn}\varsigma}{\mathrm{sn}^2\varsigma+(1+m_1)\mathrm{dn}\varsigma-1-m_1},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{m^2\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{\mathrm{sn}^2\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]+(1+m_1)\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]-1-m_1}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right],\tag{36}$$

where $m_1=\sqrt{1-m^2}$.

(21): $P=\frac{2-m^2+2m_1}{4},\quad Q=\frac{m^2+6m_1-2}{2},\quad R=\frac{2-m^2+2m_1}{4},\quad F(\varsigma)=\frac{m^2\mathrm{sn}\varsigma\mathrm{cn}\varsigma}{\mathrm{sn}^2\varsigma+(-1+m_1)\mathrm{dn}\varsigma-1-m_1},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}\right.$$
$$\left.+\delta_1\frac{m^2\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{\mathrm{sn}^2\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]+(-1+m_1)\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]-1-m_1}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{37}$$

(22): $P=\frac{C^2m^4-(B^2+C^2)m^2+B^2}{4},\quad Q=\frac{m^2+1}{2},\quad R=\frac{m^2-1}{4(C^2m^2-B^2)},$
$F(\varsigma)=\frac{\sqrt{\frac{B^2-C^2}{B^2-C^2m^2}}+\mathrm{sn}\varsigma}{B\mathrm{cn}\varsigma+C\mathrm{dn}\varsigma},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\sqrt{\frac{B^2-C^2}{B^2-C^2m^2}}+\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{B\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]+C\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{38}$$

(23): $P=\frac{B^2+C^2m^2}{4},\quad Q=\frac{1-2m^2}{2},\quad R=\frac{1}{4(B^2+C^2m^2)},\quad F(\varsigma)=\frac{\sqrt{\frac{C^2m^2+B^2-C^2}{B^2+C^2m^2}}+\mathrm{cn}\varsigma}{B\mathrm{sn}\varsigma+C\mathrm{dn}\varsigma},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\sqrt{\frac{C^2m^2+B^2-C^2}{B^2+C^2m^2}}+\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{B\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]+C\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{39}$$

(24): $P=\frac{B^2+C^2}{4},\quad Q=\frac{m^2-2}{2},\quad R=\frac{m^4}{4(B^2+C^2)},\quad F(\varsigma)=\frac{\sqrt{\frac{B^2+C^2-C^2m^2}{B^2+C^2}}+\mathrm{dn}\varsigma}{B\mathrm{sn}\varsigma+C\mathrm{cn}\varsigma},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\sqrt{\frac{B^2+C^2-C^2m^2}{B^2+C^2}}+\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{B\,\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]+C\,\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right]. \tag{40}$$

3.2. Weierstrass' elliptic function solutions

Weierstrass' elliptic function $\wp$ is defined as follows [12]:

$$\wp(z;\omega_1,\omega_2)=\frac{1}{z^2}+\sum_{m\omega_1+n\omega_2\neq0}\left\{\frac{1}{(z+m\omega_1+n\omega_2)^2}-\frac{1}{(m\omega_1+n\omega_2)^2}\right\}. \tag{41}$$

Weierstrass elliptic function solutions to (1) are derived by the employ of the solutions of (12) given in [14] as:

(25):
$g_2=\frac{4(Q^2-3PR)}{3},\quad g_3=\frac{4Q(-2Q^2+9PR)}{27},\quad F(\varsigma)=\sqrt{\frac{1}{P}\left[\wp(\varsigma;g_2,g_3)-\frac{1}{3}Q\right]},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\sqrt{\frac{1}{P}\left[\wp\left(\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right];g_2,g_3\right)-\frac{1}{3}Q\right]}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right]. \tag{42}$$

(26): $g_2=\frac{4(Q^2-3PR)}{3},\quad g_3=\frac{4Q(-2Q^2+9PR)}{27},\quad F(\varsigma)=\sqrt{\frac{3R}{3\wp(\varsigma;g_2,g_3)-Q}},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\sqrt{\frac{3R}{3\wp\left(\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right];g_2,g_3\right)-Q}}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right]. \tag{43}$$

(27): $g_2=-\frac{5QD+4Q^2+33PQR}{12},\quad g_3=\frac{21Q^2D-63PRD+20Q^3-27PQR}{216},$
$F(\varsigma)=\frac{\sqrt{12R\wp(\varsigma;g_2,g_3)+2R(2Q+D)}}{12\wp(\varsigma;g_2,g_3)+D},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\sqrt{12R\wp\left(\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right];g_2,g_3\right)+2R(2Q+D)}}{12\wp\left(\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right];g_2,g_3\right)+D}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right]. \tag{44}$$

(28): $g_2=\frac{Q^2+12PR}{12},\quad g_3=\frac{Q(36PR-Q^2)}{216},\quad F(\varsigma)=\frac{\sqrt{R}\,[6\wp(\varsigma;g_2,g_3)+Q]}{3\wp'(\varsigma;g_2,g_3)},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\sqrt{R}\left[6\wp\left(\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right];g_2,g_3\right)+Q\right]}{3\wp'\left(\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right];g_2,g_3\right)}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right]. \tag{45}$$

(29): $g_2=\frac{Q^2+12PR}{12},\quad g_3=\frac{Q(36PR-Q^2)}{216},\quad F(\varsigma)=\frac{3\wp'(\varsigma;g_2,g_3)}{\sqrt{P}\,[6\wp(\varsigma;g_2,g_3)+Q]},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{3\wp'\left(\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right];g_2,g_3\right)}{\sqrt{P}\left[6\wp\left(\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right];g_2,g_3\right)+Q\right]}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right]. \tag{46}$$

(30): $R=\frac{5Q^2}{36P},\quad g_2=\frac{2Q^2}{9},\quad g_3=\frac{Q^3}{54},\quad F(\varsigma)=\frac{Q\sqrt{-15Q/2P}\,\wp(\varsigma;g_2,g_3)}{3\wp(\varsigma;g_2,g_3)+Q},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{Q\sqrt{-15Q/2P}\,\wp\left(\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right];g_2,g_3\right)}{3\wp\left(\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right];g_2,g_3\right)+Q}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right]. \tag{47}$$

3.3. Solitons and other solutions

The soliton solutions that are going to be enumerated here come out of Jacobi's elliptic functions, listed in Section 3.1, upon performing the limiting operation when the modulus of ellipticity m approaches unity. Thus, when $m\to1^-$, the following CQ solitons, complexitons and a combination of such solutions emerge:

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\tanh\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)-\frac{2c_4\delta_1^2}{n+1}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right]. \tag{48}$$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\mathrm{sech}\left[x+\left(\frac{\alpha\kappa}{b}-\frac{c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)-\frac{c_4\delta_1^2}{n+1}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right]. \tag{49}$$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\coth\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)-\frac{2c_4\delta_1^2}{n+1}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right]. \tag{50}$$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\mathrm{csch}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2}{n+1}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right]. \tag{51}$$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\left(\coth\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right.\right.$$
$$\left.\left.\pm\,\mathrm{csch}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right)\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)-\frac{2c_4\delta_1^2}{n+1}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right]. \tag{52}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\left(\tanh\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\pm i\mathrm{sech}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right)\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)-\frac{2c_4\delta_1^2}{n+1}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{53}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+2\delta_1\mathrm{sech}\left[x+\left(\frac{\alpha\kappa}{b}-\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)-\frac{4c_4\delta_1^2}{n+1}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{54}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\sqrt{-\frac{Q}{2P}}\tanh\left(\sqrt{-\frac{Q}{2}}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{55}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\sqrt{-\frac{Q}{P}}\mathrm{sech}\left(\sqrt{Q}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{c_4\delta_1^2Q}{(n+1)P}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{56}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+2\delta_1\mathrm{csch}2\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_4\delta_1^2}{n+1}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{57}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\mathrm{sech}2\left[x+\left(\frac{\alpha\kappa}{b}-\frac{c_4n^2\beta\delta_1^2}{4b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)-\frac{c_4\delta_1^2}{n+1}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{58}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\sqrt{\frac{B^2}{B^2+C^2}}+\mathrm{sech}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)(B^2+C^2)}\right)t\right]}{B\tanh\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)(B^2+C^2)}\right)t\right]+C\mathrm{sech}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)(B^2+C^2)}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)-\frac{2c_4\delta_1^2}{(n+1)(B^2+C^2)}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{59}$$

Here, solutions (48) and (55) are QC dark solitons solutions to the governing model. QC bright solitons solutions are listed in (49), (54), (56) and (58). Eqs. (50), (51) and (57) stand for QC singular solitons solutions. Solution (52) represents QC combo singular solitons solutions. Finally, (53) is complexiton solutions.

3.4. Trigonometric function solutions

However, for $m\to 0^+$, trigonometric functions solutions and their combinations are procured as the following:

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\csc\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)-\frac{c_4\delta_1^2}{n+1}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{60}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\sec\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)-\frac{c_4\delta_1^2}{n+1}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{61}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\tan\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{2c_4\delta_1^2}{n+1}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{62}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\cot\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{2c_4\delta_1^2}{n+1}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{63}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\left(\csc\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\pm\cot\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right)\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{2c_4\delta_1^2}{n+1}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{64}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\left(\sec\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\pm\tan\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right)\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{2c_4\delta_1^2}{n+1}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{65}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\frac{\delta_1}{A}\left(\sec\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)A^2}\right)t\right]-\tan\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)A^2}\right)t\right]\right)\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{2c_4\delta_1^2}{(n+1)A^2}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right].\tag{66}$$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\sqrt{\frac{B^2-C^2}{B^2}}+\sin\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)B^2}\right)t\right]}{B\cos\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)B^2}\right)t\right]+C}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{2c_4\delta_1^2}{(n+1)B^2}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right]. \tag{67}$$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\sqrt{\frac{B^2-C^2}{B^2}}+\cos\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)B^2}\right)t\right]}{B\sin\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)B^2}\right)t\right]+C}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{2c_4\delta_1^2}{(n+1)B^2}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right]. \tag{68}$$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\frac{2\delta_1}{B\sin\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)(B^2+C^2)}\right)t\right]+C\cos\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)(B^2+C^2)}\right)t\right]}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)-\frac{4c_4\delta_1^2}{(n+1)(B^2+C^2)}+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t+\theta\right\}\right]. \tag{69}$$

4. *F*-Expansion scheme (type-II)

In this type, the solution of (10) is looked for as [16–18]

$$\psi(\varsigma)=\delta_0+\sum_{j=1}^{N}\left\{\delta_jF^j(\varsigma)+\frac{\zeta_j}{F^j(\varsigma)}\right\} \tag{70}$$

where δ_0, δ_j and ζ_j are constants that needs to be identified and $F=F(\varsigma)$ provides Eq. (12). From the balance principle, (70) becomes

$$\psi(\varsigma)=\delta_0+\delta_1F(\varsigma)+\frac{\zeta_1}{F(\varsigma)}. \tag{71}$$

Plugging (71) into (10) brings about

$$v=-\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right),$$
$$c_1=-\frac{(n-1)(c_3^2(n+1)^2\sqrt{P}-16c_4^2\delta_1^2(n+2)^2\sqrt{R})(c_3^2(n+1)^2P+4c_4^2\delta_1^2(n+2)^2(Q-2\sqrt{PR}))}{16c_4^3(n+1)(n+2)^4P^{3/2}},$$
$$c_2=-\frac{c_3(n-2)(c_3^2(n+1)^2P+2c_4^2\delta_1^2(n+2)^2(Q-6\sqrt{PR}))}{4c_4^2(n+2)^3P},$$
$$\delta_0=-\frac{c_3(n+1)}{2c_4(n+2)},\quad \delta_1=\delta_1,\quad \zeta_1=\delta_1\sqrt{\frac{R}{P}},$$
$$\omega=-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right), \tag{72}$$

where

$$\alpha=3a^2-15ab\kappa+20b^2\kappa^2,\quad \beta=a-4b\kappa. \tag{73}$$

By the use of the solution set, (1) has the formal solution as:

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1F(\varsigma)+\frac{\delta_1\sqrt{R/P}}{F(\varsigma)}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \tag{74}$$

4.1. Jacobi's elliptic function solutions

Proceeding as in type-1 of the applied scheme, elliptic functions solutions are acquired as:

(1): $P=m^2,\quad Q=-(1+m^2),\quad R=1,\quad F(\varsigma)=\mathrm{sn}\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]+\delta_1\sqrt{R/P}\,\mathrm{ns}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \tag{75}$$

(2): $P=1,\quad Q=-(1+m^2),\quad R=m^2,\quad F(\varsigma)=\mathrm{ns}\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\mathrm{ns}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]+\delta_1\sqrt{R/P}\,\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \tag{76}$$

(3): $P=1,\quad Q=-(1+m^2),\quad R=m^2,\quad F(\varsigma)=\mathrm{dc}\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\mathrm{dc}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]+\delta_1\sqrt{R/P}\,\mathrm{cd}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \tag{77}$$

(4): $P=1-m^2,\quad Q=2-m^2,\quad R=1,\quad F(\varsigma)=\mathrm{sc}\,\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\mathrm{sc}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]+\delta_1\sqrt{R/P}\,\mathrm{cs}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \tag{78}$$

(5): $P=1,\quad Q=2-m^2,\quad R=1-m^2,\quad F(\varsigma)=\mathrm{cs}\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1 cs\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]+\delta_1\sqrt{R/P}\,\mathrm{sc}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \tag{79}$$

(6): $P=\frac{1}{4},\quad Q=\frac{1-2m^2}{2},\quad R=\frac{1}{4},\quad F(\varsigma)=\mathrm{ns}\varsigma\pm cs\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\left(\mathrm{ns}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\pm cs\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)+\delta_1\sqrt{R/P}\left(\mathrm{ns}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\pm cs\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)^{-1}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \tag{80}$$

(7): $P=\frac{1-m^2}{4},\quad Q=\frac{1+m^2}{2},\quad R=\frac{1-m^2}{4},\quad F(\varsigma)=\mathrm{nc}\varsigma\pm\mathrm{sc}\,\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\left(\mathrm{nc}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\pm\mathrm{sc}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)+\delta_1\sqrt{R/P}\left(\mathrm{nc}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\pm\mathrm{sc}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)^{-1}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \tag{81}$$

(8): $P=\frac{m^2}{4},\quad Q=\frac{m^2-2}{2},\quad R=\frac{m^2}{4},\quad F(\varsigma)=\mathrm{sn}\varsigma\pm i\mathrm{cn}\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\left(\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\pm i\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)+\delta_1\sqrt{R/P}\left(\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\pm i\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)^{-1}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \tag{82}$$

(9): $P=\frac{m^2}{4},\quad Q=\frac{m^2-2}{2},\quad R=\frac{1}{4},\quad F(\varsigma)=\frac{\mathrm{sn}\varsigma}{1\pm\mathrm{dn}\varsigma},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{1\pm\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}+\delta_1\sqrt{R/P}\frac{1\pm\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \tag{83}$$

(10): $P=\frac{(1-m^2)^2}{4},\quad Q=\frac{m^2+1}{2},\quad R=\frac{1}{4},\quad F(\varsigma)=\mathrm{ds}\varsigma\pm cs\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\left(\mathrm{ds}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\pm cs\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)+\delta_1\sqrt{R/P}\left(\mathrm{ds}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\pm cs\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)^{-1}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \tag{84}$$

(11): $P>0,\quad Q<0,\quad R=\frac{m^2Q^2}{(1+m^2)^2P},\quad F(\varsigma)=\sqrt{-\frac{m^2Q}{(1+m^2)P}}\,\mathrm{sn}\left(\sqrt{-\frac{Q}{1+m^2}}\,\varsigma\right),$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\sqrt{-\frac{m^2Q}{(1+m^2)P}}\,\mathrm{sn}\left(\sqrt{-\frac{Q}{1+m^2}}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)+\delta_1\sqrt{R/P}\sqrt{-\frac{(1+m^2)P}{m^2Q}}\,\mathrm{ns}\left(\sqrt{-\frac{Q}{1+m^2}}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \tag{85}$$

(12): $P=1,\quad Q=m^2+2,\quad R=1-2m^2+m^4,\quad F(\varsigma)=\frac{\mathrm{dn}\varsigma\mathrm{cn}\varsigma}{\mathrm{sn}\varsigma},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}+\delta_1\sqrt{R/P}\frac{\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \tag{86}$$

(13): $P=\frac{A^2(m-1)^2}{4},\quad Q=\frac{m^2+6m+1}{2},\quad R=\frac{(m-1)^2}{4A^2},\quad F(\varsigma)=\frac{\mathrm{dn}\varsigma\mathrm{cn}\varsigma}{A(1+\mathrm{sn}\varsigma)(1+m\mathrm{sn}\varsigma)},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{A\left(1+\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)\left(1+m\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)}+\delta_1\sqrt{R/P}\frac{A\left(1+\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)\left(1+m\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)}{\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \tag{87}$$

(14): $P=\frac{1}{4},\quad Q=\frac{1-2m^2}{2},\quad R=\frac{1}{4},\quad F(\varsigma)=\frac{\mathrm{sn}\varsigma}{1\pm\mathrm{cn}\varsigma},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{1\pm\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}+\delta_1\sqrt{R/P}\frac{1\pm\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \quad (88)$$

(15): $P=\frac{1-m^2}{4},\quad Q=\frac{1+m^2}{2},\quad R=\frac{1-m^2}{4},\quad F(\varsigma)=\frac{\mathrm{cn}\varsigma}{1\pm\mathrm{sn}\varsigma},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{1\pm\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}+\delta_1\sqrt{R/P}\frac{1\pm\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \quad (89)$$

(16): $P=\frac{2-m^2+2m_1}{4},\quad Q=\frac{m^2+6m_1-2}{2},\quad R=\frac{2-m^2+2m_1}{4},\quad F(\varsigma)=\frac{m^2\mathrm{sn}\varsigma\mathrm{cn}\varsigma}{\mathrm{sn}^2\varsigma+(-1+m_1)\mathrm{dn}\varsigma-1-m_1},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{m^2\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{\mathrm{sn}^2\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]+(-1+m_1)\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]-1-m_1}+\delta_1\sqrt{R/P}\frac{\mathrm{sn}^2\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]+(-1+m_1)\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]-1-m_1}{m^2\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \quad (90)$$

(17): $P=\frac{B^2+C^2m^2}{4},\quad Q=\frac{1-2m^2}{2},\quad R=\frac{1}{4(B^2+C^2m^2)},\quad F(\varsigma)=\frac{\sqrt{\frac{C^2m^2+B^2-C^2}{B^2+C^2m^2}}+\mathrm{cn}\varsigma}{B\mathrm{sn}\varsigma+C\mathrm{dn}\varsigma},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\sqrt{\frac{C^2m^2+B^2-C^2}{B^2+C^2m^2}}+\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{B\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]+C\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}+\delta_1\sqrt{R/P}\frac{B\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]+C\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{\sqrt{\frac{C^2m^2+B^2-C^2}{B^2+C^2m^2}}+\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \quad (91)$$

(18): $P=\frac{B^2+C^2}{4},\quad Q=\frac{m^2-2}{2},\quad R=\frac{m^4}{4(B^2+C^2)},\quad F(\varsigma)=\frac{\sqrt{\frac{B^2+C^2-C^2m^2}{B^2+C^2}}+\mathrm{dn}\varsigma}{B\mathrm{sn}\varsigma+C\mathrm{cn}\varsigma},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\sqrt{\frac{B^2+C^2-C^2m^2}{B^2+C^2}}+\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{B\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]+C\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}+\delta_1\sqrt{R/P}\frac{B\mathrm{sn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]+C\mathrm{cn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}{\sqrt{\frac{B^2+C^2-C^2m^2}{B^2+C^2}}+\mathrm{dn}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2(Q-6\sqrt{PR})}{(n+1)P}\right)t+\theta\right\}\right]. \quad (92)$$

4.2. Solitons and other solutions

When $m\to1^-$, QC singular solitons, QC combo solitons, some solitary waves and complexitons emerge

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\tanh\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]+\delta_1\coth\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}-\frac{8c_4\delta_1^2}{n+1}\right)t+\theta\right\}\right]. \quad (93)$$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\mathrm{csch}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{c_4\delta_1^2}{n+1}\right)t+\theta\right\}\right]. \quad (94)$$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\left(\coth\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\pm\mathrm{csch}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right)+\delta_1\left(\coth\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\pm\mathrm{csch}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right)^{-1}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}-\frac{8c_4\delta_1^2}{n+1}\right)t+\theta\right\}\right]. \quad (95)$$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\left(\tanh\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\pm i\mathrm{sech}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right)+\delta_1\left(\tanh\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\pm i\mathrm{sech}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right)^{-1}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}-\frac{8c_4\delta_1^2}{n+1}\right)t+\theta\right\}\right]. \quad (96)$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\tanh\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]}{1\pm\operatorname{sech}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]}+\delta_1\frac{1\pm\operatorname{sech}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]}{\tanh\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}-\frac{8c_4\delta_1^2}{n+1}\right)t+\theta\right\}\right]. \tag{97}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+2\delta_1\sqrt{-\frac{Q}{2P}}\coth2\left(\sqrt{-\frac{Q}{2}}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)P}\right)t\right]\right)\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{4c_4\delta_1^2Q}{(n+1)P}\right)t+\theta\right\}\right]. \tag{98}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+2\delta_1\operatorname{csch}2\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}+\frac{3c_4\delta_1^2}{n+1}\right)t+\theta\right\}\right]. \tag{99}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\tanh\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]}{1\pm\operatorname{sech}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]}+\delta_1\frac{1\pm\operatorname{sech}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]}{\tanh\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}-\frac{8c_4\delta_1^2}{n+1}\right)t+\theta\right\}\right]. \tag{100}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}\right.$$
$$+\delta_1\frac{\sqrt{\frac{B^2}{B^2+C^2}}+\operatorname{sech}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)(B^2+C^2)}\right)t\right]}{B\tanh\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)(B^2+C^2)}\right)t\right]+C\operatorname{sech}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)(B^2+C^2)}\right)t\right]}$$
$$\left.+\frac{\delta_1}{B^2+C^2}\frac{B\tanh\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)(B^2+C^2)}\right)t\right]+C\operatorname{sech}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)(B^2+C^2)}\right)t\right]}{\sqrt{\frac{B^2}{B^2+C^2}}+\operatorname{sech}\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)(B^2+C^2)}\right)t\right]}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}-\frac{8c_4\delta_1^2}{(n+1)(B^2+C^2)}\right)t+\theta\right\}\right]. \tag{101}$$

Here, solution (93) is dark-singular combo optical soliton to the model. Singular solitons are listed in (94), (98) and (99). Solution (95) represents combo singular solitons solutions. Finally, Eq. (96) stands for complexiton solutions.

4.3. *Trigonometric function solutions*

Whenever $m\to0^+$, trigonometric functions solutions and their combinations are achieved as:

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\csc\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}-\frac{c_4\delta_1^2}{n+1}\right)t+\theta\right\}\right]. \tag{102}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\sec\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}-\frac{c_4\delta_1^2}{n+1}\right)t+\theta\right\}\right]. \tag{103}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\tan\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right.$$
$$\left.+\delta_1\cot\left[x+\left(\frac{\alpha\kappa}{b}+\frac{c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}-\frac{4c_4\delta_1^2}{n+1}\right)t+\theta\right\}\right]. \tag{104}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\left(\csc\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\pm\cot\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right)\right.$$
$$\left.+\delta_1\left(\csc\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\pm\cot\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right)^{-1}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}-\frac{4c_4\delta_1^2}{n+1}\right)t+\theta\right\}\right]. \tag{105}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\left(\sec\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\pm\tan\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right)\right.$$
$$\left.+\delta_1\left(\sec\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\pm\tan\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)}\right)t\right]\right)^{-1}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}-\frac{4c_4\delta_1^2}{n+1}\right)t+\theta\right\}\right]. \tag{106}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\frac{\delta_1}{A}\left(\sec\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)A^2}\right)t\right]\right.\right.$$
$$\left.-\tan\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)A^2}\right)t\right]\right)$$
$$\left.+\frac{\delta_1}{A^2}\left(\sec\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)A^2}\right)t\right]-\tan\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)A^2}\right)t\right]\right)^{-1}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}-\frac{4c_4\delta_1^2}{(n+1)A^2}\right)t+\theta\right\}\right]. \tag{107}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\delta_1\frac{\sqrt{\frac{B^2-C^2}{B^2}}+\cos\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)B^2}\right)t\right]}{B\sin\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)B^2}\right)t\right]+C}\right.$$
$$\left.+\frac{\delta_1}{B^2}\frac{B\sin\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)B^2}\right)t\right]+C}{\sqrt{\frac{B^2-C^2}{B^2}}+\cos\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)B^2}\right)t\right]}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}-\frac{4c_4\delta_1^2}{(n+1)B^2}\right)t+\theta\right\}\right]. \tag{108}$$

$$q(x,\,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\frac{2\delta_1}{B\sin\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)(B^2+C^2)}\right)t\right]+C\cos\left[x+\left(\frac{\alpha\kappa}{b}+\frac{4c_4n^2\beta\delta_1^2}{b(n+1)(B^2+C^2)}\right)t\right]}\right\}^{\frac{1}{n}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\kappa^3(a-b\kappa)+\frac{3c_3^2(n+1)}{2c_4(n+2)^2}-\frac{4c_4\delta_1^2}{(n+1)(B^2+C^2)}\right)t+\theta\right\}\right]. \tag{109}$$

5. *F*-expansion scheme (type-III)

According to this methodology, the solution of (10) is taken to be [19]

$$\psi(\varsigma) = \delta_0 + \sum_{j=1}^{N} \{\delta_j F^j(\varsigma) + \zeta_j F^{-j}(\varsigma) + \eta_j F^{j-1}(\varsigma)F'(\varsigma)\} \tag{110}$$

where δ_j, ζ_j and η_j are constants that needs to be designated and $F = F(\varsigma)$ is a solution of Eq. (12). From the balance principle

$$\psi(\varsigma) = \delta_0 + \delta_1 F(\varsigma) + \zeta_1 F^{-1}(\varsigma) + \eta_1 F'(\varsigma). \tag{111}$$

Substituting (111) into (10) gives rise to

Set 1:

$$\begin{aligned}
v &= -\left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2 \beta \delta_1^2}{b(n+1)P}\right),\\
c_1 &= -\frac{(n-1)(c_3^4(n+1)^4 P + 4c_3^2 c_4^2 \delta_1^2 (n+1)^2 (n+2)^2 Q + 16 c_4^4 \delta_1^4 (n+2)^4 R)}{16 c_4^3 (n+1)(n+2)^4 P},\\
c_2 &= -\frac{c_3(n-2)(c_3^2(n+1)^2 P + 2c_4^2 \delta_1^2 (n+2)^2 Q)}{4c_4^2(n+2)^3 P},\\
\delta_0 &= -\frac{c_3(n+1)}{2c_4(n+2)}, \quad \delta_1 = \delta_1, \quad \zeta_1 = \eta_1 = 0,\\
\omega &= -\left(\kappa^3(a - b\kappa) + \frac{c_4 \delta_1^2 Q}{(n+1)P} + \frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right),
\end{aligned} \tag{112}$$

Set 2:

$$\begin{aligned}
v &= -\left(\frac{\alpha\kappa}{b} + \frac{c_4 n^2 \beta \delta_1^2}{b(n+1)P}\right),\\
c_1 &= -\frac{(n-1)(c_3^2(n+1)^2\sqrt{P} - 16c_4^2\delta_1^2(n+2)^2\sqrt{R})(c_3^2(n+1)^2 P + 4c_4^2\delta_1^2(n+2)^2(Q - 2\sqrt{PR}))}{16c_4^3(n+1)(n+2)^4 P^{3/2}},\\
c_2 &= -\frac{c_3(n-2)(c_3^2(n+1)^2 P + 2c_4^2\delta_1^2(n+2)^2(Q - 6\sqrt{PR}))}{4c_4^2(n+2)^3 P},\\
\delta_0 &= -\frac{c_3(n+1)}{2c_4(n+2)}, \quad \delta_1 = \delta_1, \quad \zeta_1 = \delta_1\sqrt{\frac{R}{P}}, \quad \eta_1 = 0,\\
\omega &= -\left(\kappa^3(a - b\kappa) + \frac{3c_3^2(n+1)}{2c_4(n+2)^2} + \frac{c_4\delta_1^2(Q - 6\sqrt{PR})}{(n+1)P}\right),
\end{aligned} \tag{113}$$

where

$$\alpha = 3a^2 - 15ab\kappa + 20b^2\kappa^2, \quad \beta = a - 4b\kappa. \tag{114}$$

Thus, according to Set 1 and Set 2, the formal solutions of Kudryashov's model are of the form

$$q(x, t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} + \delta_1 F(\varsigma)\right\}^{\frac{1}{n}} \exp\left[i\left\{-\kappa x - \left(\kappa^3(a - b\kappa) + \frac{c_4\delta_1^2 Q}{(n+1)P} + \frac{3c_3^2(n+1)}{2c_4(n+2)^2}\right)t + \theta\right\}\right] \tag{115}$$

and

$$q(x, t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} + \delta_1 F(\varsigma) + \frac{\delta_1\sqrt{R/P}}{F(\varsigma)}\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\kappa^3(a - b\kappa) + \frac{3c_3^2(n+1)}{2c_4(n+2)^2} + \frac{c_4\delta_1^2(Q - 6\sqrt{PR})}{(n+1)P}\right)t + \theta\right\}\right]. \tag{116}$$

It should be noted that since the solutions given by (115) and (116) are the same as those in Type-I and Type-II of F-expansion scheme respectively, the solutions in this section are omitted.

6. *F*-Expansion scheme (type-IV)

According to this algorithm, (10) has the solution form given by [20]

$$\psi(\varsigma) = \delta_0 + \sum_{j=1}^{N} \{\delta_j F^j(\varsigma) + \zeta_j F^{-j}(\varsigma) + \eta_j F^{j-1}(\varsigma)F'(\varsigma) + \chi_j F^{-j}(\varsigma)F'(\varsigma)\} \tag{117}$$

where δ_j, ζ_j, η_j and χ_j are constants that needs to be identified and $F = F(\varsigma)$ is a solution of Eq. (12). The balance principle implies $N = 1$ and then

$$\psi(\varsigma) = \delta_0 + \delta_1 F(\varsigma) + \zeta_1 F^{-1}(\varsigma) + \eta_1 F'(\varsigma) + \chi_1 F^{-1}(\varsigma)F'(\varsigma). \tag{118}$$

Proceeding as in previous sections, the following results are extracted:

$$\begin{aligned}
v &= -\frac{c_4 n^2 \beta \chi_1^2 + \alpha\kappa(n+1)}{b(n+1)},\\
c_1 &= -\frac{c_4\chi_1^4(n-1)(Q^2 - 4PR)}{n+1} + \frac{c_3^2\chi_1^2 Q(n^2-1)}{2c_4(n+2)^2} - \frac{c_3^4(n-1)(n+1)^3}{16c_4^3(n+2)^4},\\
c_2 &= \frac{c_3\chi_1^2 Q(n-2)}{n+2} - \frac{c_3^3(n-2)(n+1)^2}{4c_4^2(n+2)^3},\\
\delta_0 &= -\frac{c_3(n+1)}{2c_4(n+2)}, \quad \delta_1 = \zeta_1 = \eta_1 = 0, \quad \chi_1 = \chi_1,\\
\omega &= -\frac{3c_3^2(n+1)^2 + 2c_4(n+2)^2(\kappa^3(n+1)(a - b\kappa) - 2c_4\chi_1^2 Q)}{2c_4(n+1)(n+2)^2},
\end{aligned} \tag{119}$$

where

$$\alpha = 3a^2 - 15ab\kappa + 20b^2\kappa^2, \quad \beta = a - 4b\kappa. \tag{120}$$

In this case, (1) has the formal solution as:

$$q(x, t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} + \chi_1\frac{F'(\varsigma)}{F(\varsigma)}\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\frac{3c_3^2(n+1)^2 + 2c_4(n+2)^2(\kappa^3(n+1)(a - b\kappa) - 2c_4\chi_1^2 Q)}{2c_4(n+1)(n+2)^2}\right)t + \theta\right\}\right]. \tag{121}$$

6.1. Jacobi's elliptic function solutions

Elliptic functions solutions to the model are listed as:

(1): $P = m^2$, $Q = -(1 + m^2)$, $R = 1$, $F(\varsigma) = \text{sn}\varsigma$,

$$q(x, t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} + \chi_1\frac{\text{cn}\left[x + \left(\frac{c_4 n^2\beta\chi_1^2 + \alpha\kappa(n+1)}{b(n+1)}\right)t\right]\text{dn}\left[x + \left(\frac{c_4 n^2\beta\chi_1^2 + \alpha\kappa(n+1)}{b(n+1)}\right)t\right]}{\text{sn}\left[x + \left(\frac{c_4 n^2\beta\chi_1^2 + \alpha\kappa(n+1)}{b(n+1)}\right)t\right]}\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\frac{3c_3^2(n+1)^2 + 2c_4(n+2)^2(\kappa^3(n+1)(a - b\kappa) - 2c_4\chi_1^2 Q)}{2c_4(n+1)(n+2)^2}\right)t + \theta\right\}\right]. \tag{122}$$

(2): $P = m^2$, $Q = -(1 + m^2)$, $R = 1$, $F(\varsigma) = \text{cd}\varsigma$,

$$q(x, t) = \left\{-\frac{c_3(n+1)}{2c_4(n+2)} - \chi_1(1 - m^2)\frac{\text{sd}\left[x + \left(\frac{c_4 n^2\beta\chi_1^2 + \alpha\kappa(n+1)}{b(n+1)}\right)t\right]\text{nd}\left[x + \left(\frac{c_4 n^2\beta\chi_1^2 + \alpha\kappa(n+1)}{b(n+1)}\right)t\right]}{\text{cd}\left[x + \left(\frac{c_4 n^2\beta\chi_1^2 + \alpha\kappa(n+1)}{b(n+1)}\right)t\right]}\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\frac{3c_3^2(n+1)^2 + 2c_4(n+2)^2(\kappa^3(n+1)(a - b\kappa) - 2c_4\chi_1^2 Q)}{2c_4(n+1)(n+2)^2}\right)t + \theta\right\}\right]. \tag{123}$$

(3): $P = -m^2$, $Q = 2m^2 - 1$, $R = 1 - m^2$, $F(\varsigma) = \text{cn}\varsigma$,

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-\chi_1\frac{\operatorname{sn}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\operatorname{dn}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}{\operatorname{cn}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{124}$$

(4): $P=-1,\quad Q=2-m^2,\quad R=m^2-1,\quad F(\varsigma)=\operatorname{dn}\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-m^2\chi_1\frac{\operatorname{sn}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\operatorname{cn}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}{\operatorname{dn}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{125}$$

(5): $P=1,\quad Q=-(1+m^2),\quad R=m^2,\quad F(\varsigma)=\operatorname{ns}\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-\chi_1\frac{cs\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\operatorname{ds}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}{\operatorname{ns}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{126}$$

(6): $P=1,\quad Q=-(1+m^2),\quad R=m^2,\quad F(\varsigma)=\operatorname{dc}\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\chi_1(1-m^2)\frac{\operatorname{nc}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\operatorname{sc}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}{\operatorname{dc}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{127}$$

(7): $P=1-m^2,\quad Q=2m^2-1,\quad R=-m^2,\quad F(\varsigma)=\operatorname{nc}\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\chi_1\frac{\operatorname{sc}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\operatorname{dc}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}{\operatorname{nc}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{128}$$

(8): $P=m^2-1,\quad Q=2-m^2,\quad R=-1,\quad F(\varsigma)=\operatorname{nd}\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+m^2\chi_1\frac{\operatorname{cd}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\operatorname{sd}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}{\operatorname{nd}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{129}$$

(9): $P=1-m^2,\quad Q=2-m^2,\quad R=1,\quad F(\varsigma)=\operatorname{sc}\,\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\chi_1\frac{\operatorname{dc}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\operatorname{nc}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}{\operatorname{sc}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{130}$$

(10): $P=-m^2(1-m^2),\quad Q=2m^2-1,\quad R=1,\quad F(\varsigma)=\operatorname{sd}\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\chi_1\frac{\operatorname{nd}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\operatorname{cd}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}{\operatorname{sd}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{131}$$

(11): $P=1,\quad Q=2-m^2,\quad R=1-m^2,\quad F(\varsigma)=cs\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-\chi_1\frac{\operatorname{ns}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\operatorname{ds}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}{cs\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{132}$$

(12): $P=1,\quad Q=2m^2-1,\quad R=-m^2(1-m^2),\quad F(\varsigma)=\operatorname{ds}\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-\chi_1\frac{cs\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\operatorname{ns}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}{\operatorname{ds}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{133}$$

(13): $P=\frac{1}{4},\quad Q=\frac{1-2m^2}{2},\quad R=\frac{1}{4},\quad F(\varsigma)=\operatorname{ns}\varsigma+cs\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-\chi_1\operatorname{ds}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{134}$$

(14): $P=\frac{1-m^2}{4},\quad Q=\frac{1+m^2}{2},\quad R=\frac{1-m^2}{4},\quad F(\varsigma)=\operatorname{nc}\varsigma+\operatorname{sc}\,\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\chi_1\,\mathrm{dc}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{135}$$

(15): $P=\frac{1}{4},\quad Q=\frac{m^2-2}{2},\quad R=\frac{m^2}{4},\quad F(\varsigma)=\mathrm{ns}\varsigma+\mathrm{ds}\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-\chi_1\,cs\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{136}$$

(16): $P=\frac{m^2}{4},\quad Q=\frac{m^2-2}{2},\quad R=\frac{m^2}{4},\quad F(\varsigma)=\mathrm{sn}\varsigma+i\mathrm{cn}\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-i\chi_1\,\mathrm{dn}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{137}$$

(17): $P=-\frac{1}{4},\quad Q=\frac{m^2+1}{2},\quad R=\frac{(1-m^2)^2}{4},\quad F(\varsigma)=m\mathrm{cn}\varsigma+\mathrm{dn}\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-m\chi_1\,\mathrm{sn}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{138}$$

(18): $P=\frac{(1-m^2)^2}{4},\quad Q=\frac{m^2+1}{2},\quad R=\frac{1}{4},\quad F(\varsigma)=\mathrm{ds}\varsigma+cs\varsigma,$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-\chi_1\,\mathrm{ns}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{139}$$

(19): $P=1,\quad Q=m^2+2,\quad R=1-2m^2+m^4,\quad F(\varsigma)=\frac{\mathrm{dn}\varsigma\mathrm{cn}\varsigma}{\mathrm{sn}\varsigma},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-\chi_1\left(m^2\mathrm{sn}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\mathrm{cd}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]+\mathrm{ns}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\mathrm{dc}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right)\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{140}$$

(20): $P=\frac{m^2}{4},\quad Q=\frac{m^2-2}{2},\quad R=\frac{1}{4},\quad F(\varsigma)=\frac{\mathrm{sn}\varsigma}{1+\mathrm{dn}\varsigma},$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\chi_1\,cs\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2Q)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{141}$$

6.2. Solitons and other solutions

When $m\to 1^-$, CQ dark and singular solitons, CQ combo dark-singular soliton and complexiton solutions are obtained as:

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\chi_1\,\mathrm{csch}2\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)+4c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{142}$$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-\chi_1\tanh\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{143}$$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-\chi_1\left(\coth\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]-\tanh\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right)\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)+4c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{144}$$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\chi_1\tanh\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{145}$$

$$q(x,t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\chi_1\coth\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{146}$$

$$q(x,\ t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-\chi_1\coth\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-2c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{147}$$

$$q(x,\ t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-\chi_1\operatorname{csch}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)+c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{148}$$

$$q(x,\ t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-i\chi_1\operatorname{sech}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)+c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{149}$$

$$q(x,\ t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-2\chi_1\coth2\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-6c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{150}$$

$$q(x,\ t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\chi_1\operatorname{csch}\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)+c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{151}$$

Here, Eqs. (142), (146), (147), (148), (150), (151) represent singular solitons. Solutions (143) and (145) are dark solitons. Dark-singular combo optical soliton solution is given by (144). Finally, solution (149) stands for complexiton solution to the model.

6.3. Trigonometric function solutions

However, for $m \to 0^+$, the following trigonometric functions solutions are revealed:

$$q(x,\ t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\chi_1\cot\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)+2c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{152}$$

$$q(x,\ t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-\chi_1\tan\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)+2c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{153}$$

$$q(x,\ t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-\chi_1\cot\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)+2c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{154}$$

$$q(x,\ t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\chi_1\tan\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)+2c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{155}$$

$$q(x,\ t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\chi_1\left(\tan\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]+\cot\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right)\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-4c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{156}$$

$$q(x,\ t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-2\chi_1\csc2\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-4c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{157}$$

$$q(x,\ t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}-\chi_1\csc\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{158}$$

$$q(x,\ t)=\left\{-\frac{c_3(n+1)}{2c_4(n+2)}+\chi_1\sec\left[x+\left(\frac{c_4n^2\beta\chi_1^2+\alpha\kappa(n+1)}{b(n+1)}\right)t\right]\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x-\left(\frac{3c_3^2(n+1)^2+2c_4(n+2)^2(\kappa^3(n+1)(a-b\kappa)-c_4\chi_1^2)}{2c_4(n+1)(n+2)^2}\right)t+\theta\right\}\right]. \tag{159}$$

The following table gives and exhibits a spectral display of bright, dark and singular soliton solutions that can be recoverable from the four forms of F-expansion. It also displays the combo soliton solutions, namely the dark-singular as well as the singular-singular type, that are recoverable from this algorithm. Finally, the complexiton solutions that are obtained by the schemes are also enumerated.

7. Conclusions

This paper obtained CQ optical solitons to the model that was studied with Kudryashov's law of refractive index. A variety of *F*-expansion schemes was applied. These revealed solutions in terms of Weierstrass' elliptic functions and Jacobi's elliptic functions. Subsequently, the limiting operation, when the modulus of ellipticity approached unity or zero, yielded soliton solutions and/or periodic solutions as the case may be. In addition to Jacobi's elliptic function solutions, several other forms of solutions are recovered as a byproduct of the scheme and are displayed. This includes solutions (60)-(69), which are periodic and periodic-singular solutions. Although such solutions are not applicable to optics, they are listed to gain a complete spectrum to the solution structure that evolves from the integration architecture.

The plethora of results are very useful in the field of optoelectronics where the model is applicable. A lot of further future studies can be thus conducted. Several topics including collision-induced timing jitter, four-wave mixing, stochastic perturbation and several others can be considered. This model will be later extended to birefringent fibers, DWDM topology and several other devices that will lead to additional innovative results. Some other issues that need to be addressed are conservation laws, deterministic perturbation theory, quasi-stationary solitons, intra-channel soliton-soliton interaction. Moreover, additional mathematical strategies are to be applied to these models to locate further novel results [21–30]. Thus, a lot of work is up ahead!!

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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