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In a recent paper,[1] one of the main conclusions in Ref. 2 is called erroneous and some arguments are advanced to sustain this verdict.

Precisely, the authors of Ref. 1 disagree with the statement in Ref. 2 that "the condition $\mathbf{F}_p = 0$ … cannot be satisfied within the single-mode approaches …, but requires the presence of two coupled (1, 1) and (1, –1) modes (for a circular plasma …)."

In the both cases,[1,2] the analysis starts from the calculation of the integral

$$F_X^\alpha = \oint_\alpha [(\mathbf{e}_X \cdot \mathbf{b})\mathbf{B}_0 + (\mathbf{e}_X \cdot \mathbf{B}_0)\mathbf{b} - (\mathbf{b} \cdot \mathbf{B}_0)\mathbf{e}_X] \cdot d\mathbf{S}_\alpha \qquad (1)$$

for a circular plasma and a coaxial resistive wall. The notation is the same as in Ref. 2: F_X^α is the $\mathbf{e}_X$-projection of the electromagnetic force on the current carriers inside the torus α, $\alpha = in$ or out denotes the wall sides, $\mathbf{B}_0$ is the axially symmetric part of the magnetic field $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, with $\mathbf{b}$ being the perturbation,

$$\mathbf{e}_X = \mathbf{e}_R \cos\zeta - \mathbf{e}_\zeta \sin\zeta, \qquad (2)$$

ζ is the toroidal angle in the cylindrical coordinates (R, ζ, z) related to the main axis of the torus, and $\mathbf{e}_i$ are the unit vectors.

In the *toroidal* geometry,

$$\frac{\partial \mathbf{e}_R}{\partial \zeta} = \mathbf{e}_\zeta, \quad \frac{\partial \mathbf{e}_\zeta}{\partial \zeta} = -\mathbf{e}_R, \qquad (3)$$

making, thereby, $\mathbf{e}_X$ a unit vector in a fixed direction, $\partial \mathbf{e}_X/\partial\zeta = 0$, which property is used in derivations in Ref. 2. Also,

$$d\mathbf{S}_\alpha = \mathbf{n}_\alpha R d\zeta d\ell_p, \qquad (4)$$

where $\mathbf{n}_\alpha$ is the unit normal to the *toroidal* surface S_α, $d\ell_p$ is the length element of the contour of S_α in the cross section $\zeta = \text{const}$, and

$$R = R_0 - r\cos\theta, \qquad (5)$$

with r and θ being the minor radius and the poloidal angle in that cross section centered at $R = R_0$ (see Figs. 1 and 2 in Ref. 2). By using Eqs. (2), (4), and (5) and

$$\mathbf{e}_\zeta \cdot \mathbf{B}_0 = B_0 R_0/R, \qquad (6)$$

with constant B_0 for the *toroidal* magnetic field in the plasma-wall vacuum gap, we have

$$(\mathbf{e}_X \cdot \mathbf{B}_0) d\mathbf{S}_\alpha = \left(\frac{R}{R_0} B_{0R}\cos\zeta - B_0 \sin\zeta\right) d\mathbf{S}_\alpha^{cyl}, \qquad (7)$$

where $d\mathbf{S}_\alpha^{cyl}$ is the "cylindrical" analog of $\mathbf{S}_\alpha$,

$$d\mathbf{S}_\alpha^{cyl} = \mathbf{n}_\alpha R_0 d\zeta d\ell_p. \qquad (8)$$

This closes the gap between Eq. (1) and its quasi-cylindrical consequence,

$$F_X^\alpha = r_w R_0 \oint_\alpha [B_{0\theta} b_r \sin\theta\cos\zeta - B_0 b_r \sin\zeta + (\mathbf{B}_0 \cdot \mathbf{b})\cos\theta\sin\zeta] d\theta d\zeta, \qquad (9)$$

which are Eqs. (13) and (18) in Ref. 2, respectively. It is clear that B_0 must be a constant in Eq. (9), and it was treated as such in Ref. 2. This constant was obtained from multiplication of Eqs. (4) and (6). A reader can blame us that this step has not been so meticulously explained in Ref. 2, but the simplicity of the intermediate actions (4)–(8) makes it an easy exercise if any doubt would appear.

The fact that $B_0 = \text{const}$ in Eq. (9) has been explicitly stipulated in Ref. 2. However, in Ref. 1, it was proposed that this B_0 should be replaced by $B_0 R_0/R$. No proof is given there although this (incorrect)

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substitution alters the integral in Eq. (9), which affects the subsequent derivations and the outcome in Ref. 1.

This first unjustified deviation from the procedure developed in Ref. 2 does not yet allow us to make $\mathbf{F}_p = 0$, with $\mathbf{b}$ represented by a single harmonic, but it is aggravated by another one; the use of the Resistive Wall Mode (RWM) dispersion relation [Eqs. (4) and (5) in Ref. 1],

$$\gamma\tau_w = -2m(m - nq_{pl} - 1)/(m - nq_{pl} - 1 + \kappa^{2m}), \tag{10}$$

that for $m/n = 1/1$ is reduced to

$$\gamma\tau_w = 2q_w/(1 - q_w) \tag{11}$$

since $\kappa \equiv r_{pl}/r_w$ and the safety factor $q \propto r^2$ outside the plasma.

The combination of Eq. (11) with Eq. (9) deliberately, but groundlessly modified by

$$B_0 \to B_0R_0/R, \tag{12}$$

completes derivations in Ref. 1 and gives F_X^{α} different from that in Ref. 2.

It is explained above why substitution (12) is a mistake. Another question is whether Eq. (11) should be considered, as emphatically alleged in Ref. 1, "a definite and universal relation between $\gamma\tau_m$ and the value of the safety factor q" for $m/n = 1/1$?

Neither arguments nor, at least, references are given in Ref. 1 to support this surprising statement. However, even within the ideal MHD and the quasi-cylindrical model, Eq. (11) has an extremely narrow applicability area, as illustrated by a wide spectrum of results in Sec. 27 "Resistive-Wall Mode Instability" in Ref. 3. Besides, it is well known that the ideal MHD cannot describe the main features of the observed RWM dynamics (see Refs. 4 and 5 and reviews in Refs. 6 and 7). In search of a better approach, a great variety of dispersion relations for RWMs have been proposed,[6,7] and most of them can be covered by the following formula:[7,8]

$$\gamma\tau_D = -\frac{W_{no} + W_{add}}{W_{iw} + W_{add}}, \tag{13}$$

where W_{iw} and W_{no} are the "ideal MHD" perturbed energies with and without an ideal wall, the time constant τ_D is given by Eq. (8) in Ref. 8 [originally introduced in Eq. (66) of Ref. 9], and W_{add} is the combined contribution[7,10] due to the non-MHD and non-ideal effects (for example, responsible for $\mathbf{b} \neq \nabla \times (\boldsymbol{\xi} \times \mathbf{B}_0)$ in the plasma) and the difference in the magnetic field $\mathbf{b}$ in the plasma-wall vacuum gap from the Haney–Freidberg[9] test function,

$$\mathbf{b}_{HF} = c_{no}\mathbf{b}_{no} + c_{iw}\mathbf{b}_{iw}, \tag{14}$$

with $c_{no} + c_{iw} = 1$ and $c_{iw} = c_{no}\gamma\tau_D$. Incorporation of $\mathbf{b} \to \mathbf{b}_{HF}$ into the stability tasks is described in detail in Refs. 7, 8, and 11.

If anybody wants to say "universal," Eq. (13) is much better suited for that than one of its primitive offshoots, Eq. (11). Dispersion relations with the structure of Eq. (13) and various W_{add} values often appear in studies devoted to RWMs, such as in Refs. 12–16. As explained in Refs. 7, 8, and 11, the universal form is the consequence of using $\mathbf{b} = \mathbf{b}_{HF}$ instead of properly calculated $\mathbf{b}$, and the popularity is enhanced by an easy success[15] that can be reached by almost anything[6,7] resulting in $W_{add} \neq 0$. Just note that Eq. (13) turns into an equation for W_{add}, if τ_D is given (as in Refs. 12 and 14), γ is adjusted to experimental data, and both W_{iw} and W_{no} are evaluated somehow. Thus found W_{add} can be attributed to a number of mechanisms by means of one or two fitting parameters, while the mentioned and similar models contain them much more. Such a "success" does not solve the problem without further search for real physics behind W_{add} and even can be misleading, when overvalued as in Ref. 15, but it shows that, at least, Eq. (13) may be a good option, though with yet unknown W_{add}.

In contrast, Eq. (11) cannot be matched with or supported by experimental results. Irrespective of the plasma pressure, it gives unphysical $\gamma\tau_w = -2$ with the wall at infinity. The excuse[1] that "the mode $m/n = 1/1$ exists only in the external kink mode stability gap $q_{pl} < \kappa^2$ with the ideal wall at r_w" is clearly incompatible with the observation[17] that "a comprehensive disruption database of JET tokamaks,[18] containing thousands of cases, clearly indicates that the sideway forces are result of a $m/n = 1/1$ mode, well distinguished from the $m/n = 2/1$ mode, which is invisible in measurements." Also, Eq. (10) positioned as universal in Ref. 1 does not allow pressure-driven RWMs, while they are excited in experiments.[6,19–21]

To summarize, Eq. (11) is a product of oversimplified modeling. In reality, γ is different, which destroys the proof in Ref. 1 irrespective of Eq. (12). Equation (9) has a purely electromagnetic origin and is valid at any γ that appeared and was considered as a parameter in Ref. 2. The demonstrated careful inspection confirms that $B_0 = \text{const}$ in Eq. (9). Then, erroneous are the mentioned arguments and conclusions in Ref. 1, but not the contested approach and the coupled kink mode requirement in Ref. 2.

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