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A family of fuzzy multi-criteria sorting models FTOPSIS-Sort: Features, case study analysis, and the statistics of distinctions

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ABSTRACT

A family of fuzzy multi-criteria sorting models, FTOPSIS-Sort, as a fuzzy extension of Multi-Criteria Decision Analysis (MCDA) ordinary method TOPSIS, is introduced and analyzed. Models from this family differ by approaches to determining functions of fuzzy numbers (approximate computations, standard fuzzy arithmetic, and transformation method) and by methods for ranking of fuzzy numbers (two defuzzification based ranking methods are considered). The features of developing and adjusting Fuzzy TOPSIS (FTOPSIS) models to sorting problematic are presented. The developed models are implemented in the case study on a healthcare supply chain alternative selection problem. For exploring distinctions in sorting alternatives by FTOPSIS-Sort models, the special algorithms have been developed along with their integrating with Monte Carlo simulation of a large number of input scenarios, each of which is a separate (and independent of the others) multicriteria problem on sorting alternatives. The results of such an analysis demonstrate a significant distinction in sorting alternatives by different FTOPSIS-Sort models. The latter has theoretical, methodological, and applied significance within the use of Fuzzy TOPSIS (Fuzzy MCDA) sorting models.

1. Introduction

Within Multi-Criteria Decision Analysis (MCDA), sorting methods are designed to assign alternatives to one of the predefined ordered classes/categories. Multicriteria sorting alternatives under uncertainty forms an emerging trend in the decision analysis, taking into account its relevance for solving a wide range of applied problems, including processing and classification of a large amount of alternatives (Doumpos & Zopounidis, 2018; Lopez, 2019; Malczewski & Jankowski, 2020).

First ordinary multi-criteria sorting method proposed to the literature is ELECTRE-TRI (Roy, 1996); other frequently used MCDA sorting methods are UTADIS, ELECTRE TRI-C, Flow-Sort and others (Alvarez et al., 2021) (the survey of Sorting MCDA methods is presented in Section 3.1).

Models for multi-criteria sorting in fuzzy environment (FMCDA sorting models) are, as a rule, fuzzy extensions of ordinary MCDA sorting methods and have been intensively developed over the past decade (Section 3.2). Any FMCDA model implies implementation of an approach to assessing functions of fuzzy quantities (Fuzzy Numbers, FNs) along with the use of a fuzzy ranking method. In this paper, the ordinary (type-1) fuzzy sets (Zadeh, 1965) are considered. The extension of real function to function of FNs is based on the extension principle by Zadeh (Wang et al., 2009; Zadeh, 1975). However, direct implementation of the extension principle is ineffective even for assessing simple functions of FNs. For determining functions of FNs, the following *main approaches* have been used in the specialized publications: approximate assessing functions based on the basic type of FNs: triangular FNs (TrFNs) and Trapezoidal FNs (TpFNs) (Lee, 2005) (within this approach, Tr/TpFNs are propagated through all computations); Standard Fuzzy Arithmetic (SFA) (Dubois & Prade, 1978; Hanss, 2005), that utilizes operations with α-cuts for determining functions of FNs (in this case, with the exception of single works, FNs are considered as independent (Hanss, 2005; Yatsalo, Korobov, et al., 2022)); Transformation Methods (TMs): the set of numerical methods (Reduced TM, General TM, Extended TM) (Hanss, 2005), which lead to

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proper assessing functions of FNs (with increase of the used number of α-cuts) when there are dependent variables (or also in the case of non-monotonicity of real function, which is extended to function of FNs). It should be stressed, the use of approximate computing based on the basic type of FNs is a generally accepted approach in FMCDA.

Ranking of FNs is one of the key stages in FMCDA. There exist several dozens of fuzzy ranking methods, which can be used within FMCDA models (Kahraman & Tolga, 2009; Wang & Kerre, 2001a, 2001b; Wang et al., 2009; Yatsalo, Korobov, et al., 2022; Yatsalo & Martínez, 2018). In this paper, we focus on two well-known and widely used ranking methods: Centroid Index (CI) (or Center of Gravity) (Ostergaard, 1976; Yager, 1981), and Integral of Means (IM) (Yager, 1980).

To the best of our knowledge, there are no FMCDA sorting proposals, where SFA or TMs are consistently used for proper assessing functions of FNs. According to the authors of this article, there are two key reasons for this situation.

One of the reasons is that the implementation of approximate computations (using triangular (TrFNs) and/or trapezoidal (TpFNs) FNs) is elementary and can be easily implemented within a case study on fuzzy modeling. At the same time, there is also another reason, which is based on the concept of propagating a soft approach to calculations in fuzzy modeling based on granulating (input and output) information along with the thesis about the redundancy of exact calculations when using fuzzy quantities (Zadeh, 1975). However, in addition to the indicated above, assessing functions of FNs is based on the Zadeh's extension principle (Klir & Yuan, 1995; Zadeh, 1975). The exploration of differences of these two concepts forms the subject of R&Ds by the team of authors within FMCDA ranking (Yatsalo, Korobov, & Martínez, 2021; Yatsalo, Korobov, Oztaysi, et al., 2021; Yatsalo, Radaev, & Martínez, 2022) and FMCDA sorting (this paper) alternatives.

Correspondingly, the idea of this contribution is to explore how different approaches to computing functions of FNs and the choice of a fuzzy ranking method affect the results on sorting alternatives.

Taking into account the main idea of this contribution, the purpose of this work is to develop a family of FTOPSIS-Sort models using several approaches to estimating functions of FNs, as well as several methods for ranking of FNs, and explore the distinctions, including the statistics of distinctions, in sorting alternatives by these models.

The novelty of this contribution is as follows:

- A family of original models, FTOPSIS-Sort, for fuzzy multicriteria sorting of alternatives based on a fuzzy extension of the TOPSIS method is proposed;
- FTOPSIS-Sort models include those based on approximate assessing functions of FNs, as well as on SFA and TMs methods;
- Models with different ranking methods (Centroid Index and Integral of Means) are considered;
- FNs of the general form can be used as input values;
- Both crisp and fuzzy limiting profiles of the general form can be used;
- The analysis of distinctions in sorting alternatives by FTOPSIS-Sort models within the case study on healthcare supply chain alternative selection problem was carried out;
- Exploring the statistics of distinctions in sorting alternatives by developed FTOPSIS-Sort models with the use of Monte Carlo simulation of input scenarios was implemented.

There are several studies in the literature that compare the FMCDA sorting models, e.g., Campos et al. (2015), Costa et al. (2009), Liu et al. (2019), Remadi and Frikha (2019). It should be stressed, possible distinctions in ranking/sorting alternatives by different MCDA/FMCDA models (e.g., comparison of ranking/sorting alternatives by TOPSIS and PROMETHEE) can be considered as natural because they represent *different models of decision analysis/ decision making*. Exploring distinctions between FMCDA models for ranking alternatives, which are fuzzy extension of the same MCDA method, are considered in our works (Yatsalo & Korobov, 2021; Yatsalo, Korobov, et al., 2022; Yatsalo, Radaev, & Martínez, 2022).

The paper is organized as follows. Section 2 revises the basic notions used in this paper. In Section 3, a brief survey of ordinary and fuzzy multi-criteria sorting models is presented. In Section 4, the family of FTOPSIS-Sort models is introduced. The use of FTOPSIS-Sort models in the case study on healthcare supply chain alternative selection problem is considered in Section 5. In Section 6, the distinction analysis on sorting alternatives by different FTOPSIS-Sort models is explored based on Monte Carlo simulating input scenarios, and Section 7 concludes this paper.

2. Preliminaries

This section provides definitions of the basic concepts and terms regarding fuzzy numbers, as well as methods for ranking of fuzzy numbers used in this paper.

2.1. Fuzzy numbers

A fuzzy set A (Zadeh, 1965) defined on the universal set X is an extension of the classical (crisp) set, in which the membership function, $\mu_A(x)$, of an element $x \in X$, can take any values in the closed interval (segment) [0, 1]. The support of a fuzzy set A on the universal set X is the crisp set $supp(A) = \{x \in X : \mu_A(x) > 0\}$; the kernel of a fuzzy set A is a crisp set $ker(A) = \{x \in X : \mu_A(x) = 1\}$; A is a normal fuzzy set, if $ker(A) \neq \emptyset$; a fuzzy set A on $\mathbb{R}$ is bounded if $supp(A) \subseteq [a, b]$, where a and b are real numbers, $a \leq b$, and $[a, b]$ is a segment. The alpha-cut (α-cut) of a fuzzy set A, $\alpha \in (0, 1]$, is the crisp set $A_\alpha = \{x \in X : \mu_A(x) \geq \alpha\}$ (Dubois & Prade, 1978; Hanss, 2005).

There are several approaches to the definition of fuzzy numbers (FNs) (Dubois & Prade, 1978; Hanss, 2005; Klir & Yuan, 1995; Lee, 2005; Stefanini et al., 2008; Wang et al., 2009). The following definition, which is based on the α-cuts approach, is the most common.

Definition 1 (*Klir & Yuan, 1995*). A fuzzy number Z is a normal bounded fuzzy set on $\mathbb{R}$ with the following property: for each $\alpha \in (0, 1]$, α-cut Z_α is a closed interval.

It should be noted that an α-cut consisting of a single point is a segment by definition.

Throughout the paper, $\mathbb{F}$ is a set of FNs according to Definition 1.

Let $[A_0, B_0] = closure\{[A_\alpha, B_\alpha], \alpha \in (0, 1]\}$, where $[A_\alpha, B_\alpha] = Z_\alpha$. Then FN Z can be identified with the family of segments (Klir & Yuan, 1995; Wang et al., 2009):

$$Z = \{[A_\alpha, B_\alpha], \alpha \in [0, 1]\}, \tag{1}$$

To simplify the description in the process of using operations with FNs, let us agree to call hereafter the segment $[A_0, B_0]$ of FN (1) as the α-cut for $\alpha = 0$. A FN $Z = \{[A_\alpha, B_\alpha]\}$ is positive if $A_0 > 0$, and non-negative if $A_0 \geq 0$.

Functions of FNs as an extension of real functions are defined based on the Zadeh's extension principle (Zadeh, 1975). Let $G \subseteq \mathbb{R}^n$, $f : G \to \mathbb{R}$ be a real function, and propagation of fuzziness by function $f(x_1, \ldots, x_n)$ is implemented, i.e., instead of real numbers x_i, FNs Z_i are used, and the membership function, μ_Z, of fuzzy quantity $Z = f(Z_1, \ldots, Z_n)$, is determined with application of the extension principle (Lee, 2005; Wang et al., 2009; Zadeh, 1975):

$$\mu_Z(z) = \bigvee_{z=f(x_1,\ldots,x_n)} (\bigwedge_{i=1,\ldots,n} \mu_{Z_i}(x_i)); \tag{2}$$

and $\mu_Z(z) = 0$ if $f^{-1}(z) = \emptyset$.

For determining functions of FNs, the following main approaches are used.

- *Implementation of the Zadeh's Extension Principle (EP)* (Wang et al., 2009; Zadeh, 1975); in practice, the EP is not effective even for assessing simple arithmetic operations. Hereafter, the term *proper* estimation of a function of FNs is used if the output result coincides with that based on the EP.
- *Standard Fuzzy Arithmetic (SFA)* (Dubois & Prade, 1978; Hanss, 2005), it utilizes operations with marginal values of α-cuts for determining functions of FNs (which are considered as independent ones); SFA can often lead to *overestimation* of the output result in comparison with the proper value, assessed based on the EP, when there are dependent variables (Hanss, 2005; Yatsalo, Korobov, & Martínez, 2021; Yatsalo, Korobov, et al., 2022) in the expression under estimation. It should be also stressed, when evaluating a function of FNs, $f(Z_1, \dots, Z_n)$, for a non-monotonic (with respect to some arguments x_j) real function $f(x_1, \dots, x_n)$, the use of SFA can lead to an *underestimation* of the output value (Radaev et al., 2022).
- *Approximate assessing functions* (along with input basic types of FNs: triangular FNs (TrFNs) or/and Trapezoidal FNs (TpFNs)); within such an approach, all functions of Tr/TpFNs result in Tr/TpFNs. It should be emphasized, approximate computing based on Tr/TpFNs constitutes a narrowing of SFA to the use of two segments (their marginal points) for all FNs under consideration: the α-cuts for $\alpha = 1$ and $\alpha = 0$. The use of approximate computing is a generally accepted approach in FMCDA.
- Transformation Methods (TMs): the set of numerical methods (Reduced TM, General TM, Extended TM) (Hanss, 2005), which lead to proper assessing functions of FNs (with increasing the used number of α-cuts) when there are dependent variables; the technology of GTM/ETM can also be implemented for assessing the output result for non-monotonic functions $f(x_1, \dots, x_n)$ (including the cases without dependent variables in corresponding expressions). TMs are variants of numerical methods for interval computing (Dawood, 2011; Nguyen et al., 2012). In the same time, TMs often require a significant effort taking into account the implementation of corresponding algorithms and time of computing. Below, when TMs are used, we will call the output FNs as proper ones (despite a limited number of α-cuts used in computations).

As far as the authors know, there are no FMCDA proposals, where SFA or TMs are *consistently used for proper assessing functions of FNs of the general type* (the exception are our works (Yatsalo, Korobov, & Martínez, 2021; Yatsalo et al., 2020; Yatsalo, Korobov, et al., 2022; Yatsalo, Radaev, & Martínez, 2022)).

2.2. Ranking of fuzzy numbers

Two main classes of fuzzy ranking methods used in FMCDA are defuzzification based and pairwise comparison ones (Kahraman & Tolga, 2009; Wang & Kerre, 2001a, 2001b; Wang et al., 2009). In this paper, two defuzzification ranking methods are used: Centroid index, and Integral of Means.

Centroid Index, CI (Center of Gravity, or Yager-1 - with $supp(Z) \in [0,1]$) (Ostergaard, 1976; Wang & Kerre, 2001a; Wang et al., 2009; Yager, 1981), is based on the following expression:

$$CI(Z) = \int x\,Z(x)dx \Big/ \int Z(x)dx, \tag{3}$$

here, $Z(x) = \mu_Z(x)$; for singleton $Z = c$, $CI(Z) = c$.

Within Integral of Means, IM (or Yager-2) (Wang & Kerre, 2001a; Wang et al., 2009; Yager, 1980), the following value for FN $Z = \{[A_\alpha, B_\alpha]\}$ is determined:

$$IM(Z) = \int_0^1 (A_\alpha + B_\alpha)/2 \; d\alpha. \tag{4}$$

FN with higher value of CI (IM) has higher rank according to CI (IM) ranking method.

Comparison of CI and IM ranking methods as well as the dubious advantages of CI over IM are discussed in Yatsalo, Korobov, et al. (2022), Yatsalo, Radaev, and Martínez (2022). The fulfillment of basic requirements (axioms) to fuzzy ranking by CI and IM have been explored in Wang and Kerre (2001a) and (with the analysis in depth) in Yatsalo, Korobov, and Martínez (2021).

3. A brief survey of multi-criteria sorting models

In this section, a brief survey of ordinary and fuzzy multicriteria sorting methods is presented. Through the paper, the term *method* is utilized within MCDA (e.g., TOPSIS method); the term *model* is used within FMCDA (FTOPSIS model).

3.1. Ordinary multi-criteria sorting methods

Table 1 presents the ordinary multi-criteria sorting methods with some details: the developers, representative publications, and total number of publications using the method. First ordinary multi-criteria sorting method proposed to the literature is ELECTRE-TRI (Roy, 1996) and has the maximum number of publications in the literature, 247, that constitutes about 63% of the whole studies on this topic (all data are for Sept 2021). The other frequently used MCDA sorting methods are UTADIS (13%), ELECTRE TRI-C (5%), and Flow-Sort (4%), respectively. Some of the most common applications areas of ordinary multi-criteria sorting methods are risk assessment, performance analysis, project selection, and supplier selection. A comprehensive survey of MCDA sorting methods and their classification is presented in Alvarez et al. (2021), Martinez et al. (2023).

3.2. Fuzzy multi-criteria sorting models

Table 2 presents the fuzzy multi-criteria sorting models with some details: the developers, the type of fuzzy sets and numbers used, whether approximate computations are used or not, the type of ranking method used within the model, and the areas of application.

A comprehensive survey of FMCDA sorting models, their application and comparison is also presented in the book (Martinez et al., 2023).

The FMCDA sorting studies mainly concentrated on fuzzy Flow-Sort, fuzzy ELECTRE TRI, and fuzzy AHPSort models. About 50% of these studies applied ordinary fuzzy sets, which is followed by interval type-2 fuzzy sets and intuitionistic fuzzy sets by 20% each. After a comprehensive literature search, to the best knowledge of the authors, there are no fuzzy multi-criteria sorting models that apply standard fuzzy arithmetic or transformation methods for determining functions of FNs. Some of the most common applications are supplier selection and risk evaluation.

There are a few studies in the literature that compare FMCDA sorting models. Table 3 shows the comparative fuzzy multi-criteria sorting studies with their comparison style and the application areas.

4. Methodology: FTOPSIS-Sort models

In this section, fuzzy extensions of ordinary MCDA method TOPSIS to a family of FMCDA models with the common name FTOPSIS are presented along with adjusting these models to a family of multicriteria sorting models FTOPSIS-Sort.

Table 1
Ordinary MCDA sorting methods.

Ordinary MCDA-sorting methods	Developer author(s) (Year)	Representative publications	Total number of publications
AHP-Sort	Ishizaka et al. (2012)	Performance analysis of offshore providers (Ishizaka & López, 2018), Performance of recommender system (Toledo et al., 2019), COVID problematic (Liu et al., 2022)	12
Choquet Integral (for sorting problems)	Choquet (1954)	Robust ordinal regression applied to Choquet integral for sorting problems (Greco et al., 2010), Comparative analysis of Choquet Integral and Flow-Sort methods for choosing supplier management strategy (Sepúlveda, 2015), Sorting countries into groups based on income distributions (Karsu, 2016), Evaluating efficiency of the query generation strategy (Benabbou et al., 2017)	4
ELECTRE-Sort	Ishizaka and Nemery (2014)	Assigning machines to incomparable maintenance strategies (Ishizaka & Nemery, 2014)	1
ELECTRE-TRI (ELECTRE TRI-B)	Roy (1996)	Problem of sorting in water distribution networks (Trojan et al., 2023), Risk sorting of natural gas pipelines (Brito et al., 2010), Photovoltaic solar farms site selection (Sánchez-Lozano et al., 2014)	247
ELECTRE TRI-C	Almeida-Dias et al. (2010)	Assigning priority classes to activities in project management (Mota & de Almeida, 2012), Risk assessment in agricultural areas (Macary et al., 2013)	19
ELECTRE TRI-nB	Fernández et al. (2017)	Evaluation of R&D projects (Fernández et al., 2019)	3
ELECTRE TRI-nC	Almeida-Dias et al. (2012)	Identifying favorable climates for tourism (Mailly et al., 2014), Assessing governance capacities on energy efficiency (Cabeça et al., 2021)	13
Flow-Sort	Nemery and Lamboray (2008)	Evaluating logistics services providers (Sepúlveda, 2013), Sorting mutual funds with respect to process-oriented social responsibility (Verheyden & De Moor, 2014), Supplier performance appraisal in supply chains (Sepúlveda & Derpich, 2014)	15
TOPSIS-Sort	Sabokbar et al. (2016)	Sort corporate bonds based on financial statements and expert's assessment (Muhsen et al., 2018), Sorting tourist sites for perceived COVID-19 exposure (Yamagishi & Ocampo, 2021)	5
FS-GAIA	Nemery et al. (2012)	Assessment of innovation performances of small and medium enterprise (Nemery et al., 2012)	1
MACBETH-Sort	Ishizaka and Gordon (2017)	Assigning access and entrance solutions in ABC classes (Ishizaka & Gordon, 2017)	1
MHDIS (Multi-group Hierarchical DIScrimination)	Doumpos et al. (2001)	Country risk assessment (Doumpos & Zopounidis, 2002), Credit rating of Asian banks (Pasiouras, Gaganis, & Doumpos, 2007), Identification of acquisition targets in EU banking industry (Pasiouras, Tanna, & Zopounidis, 2007)	4
PROAFTN (PROcédure d' Affectation Floue dans le cadre de la problématique de Tri Nominal)	Belacel (2000)	ABC inventory classification (Douissa & Jabeur, 2016), Medical diagnosis (Belacel & Cuperlovic-Culf, 2019; Belacel et al., 2005)	12
PROMSORT	Araz and Ozkarahan (2005)	Supplier selection (de Oliveira e Silva et al., 2015; Gonçalo & Alencar, 2014)	5
UTADIS (UTilités Additives DIScriminantes)	Zopounidis and Doumpos (1999)	Project selection (Karasakal & Aker, 2017), Risk evaluation (Laryea et al., 2020; Lou et al., 2010; Ulucan & Atici, 2013)	52

4.1. Fuzzy extension of TOPSIS

MCDA method TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) (Hwang & Yoon, 1981) has been extended to fuzzy models in many publications, e.g., Chen (2000), Chen and Hwang (1992), Kaya and Kahraman (2011), Yatsalo et al. (2020). Comprehensive survey with statistical data on FTOPSIS usage has been presented in Kahraman et al. (2015), Yatsalo et al. (2020).

The generalized criterion (coefficient of closeness) for TOPSIS within a multi-criteria problem with n alternatives and m criteria is presented as follows:

$$D_i = \frac{D_i^-}{D_i^- + D_i^+} = \frac{(\sum_1^m w_k^2 (x_{ik} - x_k^-)^2)^{1/2}}{(\sum_1^m w_k^2 (x_{ik} - x_k^-)^2)^{1/2} + (\sum_1^m w_k^2 (x_k^+ - x_{ik})^2)^{1/2}}, \quad (5)$$

here x_{ik} is a normalized criterion value of alternative a_i for criterion k, $i = 1, \dots, n$, $k = 1, \dots, m$, x_k^+ and x_k^- are, respectively, normalized coordinates of *ideal* and *anti-ideal* points/alternatives in $\mathbb{R}^m$, w_k is a weight coefficient, D_i^+ and D_i^- are, correspondingly, weighted distances from the alternative $a_i = (x_{i1}, \dots, x_{im})$ to ideal, $I^+ = (x_1^+, \dots, x_m^+)$, and anti-ideal, $I^- = (x_1^-, \dots, x_m^-)$, alternatives.

A general approach to fuzzy extension of TOPSIS has been presented in Yatsalo, Korobov, and Martínez (2021), Yatsalo et al. (2020), Yatsalo, Radaev, and Martínez (2022). Any fuzzy extension of an MCDA methods is based on the use of fuzzy criteria values, c_{ij}, fuzzy weight coefficients, w_j, along with a fuzzy ranking method.

There are several approaches to normalization of source criteria values, c_{ij}, of alternative a_i for criterion j, $i = 1, \dots, n$, $j = 1, \dots, m$, within ordinary TOPSIS (Chen, 2000; Hwang & Yoon, 1981; Yatsalo et al., 2020). The influence of (standard non-linear) local and global

Table 2
Fuzzy multi-criteria sorting models.

Author (Year)	Fuzzy multi-criteria sorting models	Type of fuzzy sets	Type of fuzzy numbers used	Approximate computations	Type of ranking method	Application area
Krejčí and Ishizaka (2018)	FAHPSort	Ordinary fuzzy sets	TrFNs	Yes; along with the Constrained Fuzzy Arithmetic	CI, Distance criterion (dissemblance index)	Tourism problem
Xu et al. (2019)	IT2FSs AHPSort II	Interval type-2 fuzzy sets	IT2TrFNs	Yes	KM algorithm	Sustainable supplier selection
Mei et al. (2019)	AHPSort II	Ordinary fuzzy sets	TrIFNs	Yes	Aggregation operator	Internet public opinion risk grading
Liu et al. (2019)	BWMSort II	Interval type-2 fuzzy sets	IT2FNs	Yes	Best-to-others vector (BTO) and the others-to-worst vector (OTW)	Internet public opinion risk grading
Ouhibi and Frikha (2020)	F-CODAS-Sort	Ordinary fuzzy sets	TpFNs	Yes	Euclidean and Taxicab distances	Evaluating environmental quality
Galo et al. (2018)	ELECTRE TRI	Hesitant fuzzy sets	TrHFNs	Yes	Credibility index	Supplier categorization
Govindan and Jepsen (2016)	ELECTRE TRI-C	Intuitionistic fuzzy sets	TrIFNs	Yes	λ-cut	Supplier risk assessment
Pereira, de Oliveira, Gomes, and Araújo (2019)	ELECTRE TRI-C	Ordinary fuzzy sets	TpFNs	Yes	Concordance index	Sorting retail locations in a large urban city
Pereira, de Oliveira, Morais, Costa, and Arroyo-López (2019)	ELECTRE TRI-C	Hesitant fuzzy sets	TpFNs	Yes	Score and a deviation functions	Supplier development
Campos et al. (2015)	F-FlowSort	Ordinary fuzzy sets	TrFNs	Yes	CI	Exploitation of the low enthalpy geothermal area
Pelissari, Ben-Amor, and de Oliveira (2019)	Fuzzy-FlowSort	Ordinary fuzzy sets	TrFNs	Yes	CI	Pharmaceutical suppliers classification
Pelissari, Oliveira, Ben Amor, and Abackerli (2019)	Fuzzy-FlowSort	Ordinary fuzzy sets	TrFNs	Yes	CI	Supplier selection
Remadi and Frikha (2019)	IFS-FlowSort	Intuitionistic fuzzy sets	TrIFNs	Yes	Defuzzification by Gani and Abbas (2014) operator	Illustrative numerical example
Remadi and Frikha (2020)	IFS-FlowSort	Intuitionistic fuzzy sets	IFNs	Yes	Defuzzification by Gani and Abbas (2014) operator	Green supplier evaluation
Moheimani et al. (2021)	IT2TrF Flowsort	Interval type-2 fuzzy sets	IT2TrFNs	Yes	Distance-based approach proposed by Chen (2014)	Assessing agility of hospitals in disaster management

Table 3
Comparisons of fuzzy multi-criteria sorting methods.

Author (Year)	Compared fuzzy MCDA sorting methods	Style of comparison	Application area
Campos et al. (2015)	FlowSort and F-FlowSort	Case study	Exploitation of the low enthalpy geothermal area
Remadi and Frikha (2019)	FlowSort, F-FlowSort and Intuitionistic fuzzy FlowSort	Case study	Illustrative numerical example
Liu et al. (2019)	AHPSort II and BWMSort II	Case study	Illustrative numerical example

(linear) approaches to criteria normalization on ranking alternatives has been analyzed in Yatsalo, Radaev, and Martínez (2022). In this work, normalization of criteria values in FTOPSIS models, $c_{ij} \to x_{ij}$ (c_{ij} and x_{ij} are here FNs of the general form) is based on the following linear expressions (Yatsalo, Korobov, & Martínez, 2021; Yatsalo et al., 2020; Yatsalo, Radaev, & Martínez, 2022):

for *benefit criterion* j:

$$x_{ij} = (c_{ij} - A_0^j)/(B_0^j - A_0^j), \tag{6}$$

for *cost criterion* j:

$$x_{ij} = (B_0^j - c_{ij})/(B_0^j - A_0^j), \tag{7}$$

here, $c_{ij} = \{[A_\alpha^{ij}, B_\alpha^{ij}]\}$, $i = 1,\dots,n$, $j = 1,\dots,m$, $\alpha \in [0,1]$, $A_0^j = \min_i A_0^{ij}$, $B_0^j = \max_i B_0^{ij}$. Within the *global* linear approach to normalization of criteria values, A_0^j and B_0^j in (6) and (7) are substituted by values $X_0^{j,min}$ and $X_0^{j,max}$ such that: $X_0^{j,min} \le A_0^j \le B_0^j \le X_0^{j,max}$, $j = 1,\dots,m$; $X_0^{j,min}$ and $X_0^{j,max}$ are considered as potential minimal and maximal marginal points of supports for FNs, which can be used for criterion j within the specific applied problem under consideration.

In accordance with (6) and (7), $supp(x_{ij}) \subseteq [0,1]$, and all normalized criteria (in dimensionless x-scale) are benefit ones. Ideal and anti-ideal points are set, correspondingly, as $I^+ = (1,\dots,1)$, $I^- = (0,\dots,0)$.

Within FTOPSIS models, as a rule, the generalized criterion $D(a_i)$ (5) for alternative a_i, $i = 1,\dots,n$, is utilized. There is also another approach to assessing the generalized criterion for FTOPSIS (Yatsalo, Korobov, & Martínez, 2021; Yatsalo et al., 2020; Yatsalo, Radaev, & Martínez, 2022):

$$D_i = D(a_i) = \frac{1}{1 + D_i^+/D_i^-}. \tag{8}$$

For ordinary TOPSIS methods, expressions (5) and (8) are equivalent. When using FTOPSIS models with proper assessing functions of FNs, e.g., with the use of TMs, FNs D_i (5) and (8) coincides (and are proper values). However, for FTOPSIS models with implementation of SFA and approximate computations, which assumes independence of all FNs in nominator and denominator, Eqs. (5) and (8) are not equivalent: FN D_i (5) has a greater overestimation (Hanss, 2005) in comparison with D_i (8) (Yatsalo, Korobov, & Martínez, 2021; Yatsalo et al., 2020). There are also other significant differences between the models with evaluating a generalized criterion based on (5) and (8) (see Section 5.2, Figs. 1 and 2).

In this paper, FTOPSIS models differ by approaches for assessing functions of FNs based on expressions (5) and (8) and by methods for ranking output values of the generalized criterion, CI and IM.

Assigning weight coefficients in TOPSIS can be based on one of the existing approaches, including the use of subjective and objective weighting (Deng et al., 2000; Olson, 2004).

A family of FTOPSIS models has been presented and analyzed in our works (Yatsalo, Korobov, & Martínez, 2021; Yatsalo et al., 2020; Yatsalo, Radaev, & Martínez, 2022). In this paper, the following FTOPSIS models are considered:

- Models FTTrCI and FTTpCI, and FTTrIM and FTTpIM: hereinafter, for the convenience of forming abbreviated model names, FT means FTOPSIS; Tr/Tp means TrFNs/TpFNs along with *approximate computations* based on TrFNs/TpFNs (i.e., with propagating TrFNs/TpFNs through all calculated formulas of the model); CI and IM are methods of ranking output FNs (Section 2.2) D_i, $i = 1,\dots,n$, for generalized criterion (5) or (8), depending on the used scenario. These models lead to *overestimation* of the generalized criterion value (5) and (8) (Yatsalo, Korobov, & Martínez, 2021; Yatsalo et al., 2020). It can be added, FTOPSIS model with the use of approximate method with propagating triangular/trapezoidal FNs along with CI ranking is the most popular in applications;
- Models FTSCI, FTSIM: along with the denotations indicated above, S means here SFA (Standard Fuzzy Arithmetic), i.e., assessing D_i, $i = 1,\dots,n$, (5)/(8) is based on SFA. It should be stressed, implementation of SFA also leads to *overestimation* of the output value in comparison with the proper value of D_i (Yatsalo, Korobov, & Martínez, 2021; Yatsalo et al., 2020);
- Models FTRCI, FTRIM: R means here the use of RTM (Reduced Transformation Method) when assessing D_i (5)/(8); these models result in proper values D_i, $i = 1,\dots,n$, (Yatsalo et al., 2020).

4.2. The family of fuzzy multicriteria sorting models FTOPSIS-Sort

Within MCDA/FMCDA sorting models, ordered categories, $Q_1,\dots,Q_K$, are used to sort alternatives under consideration, where category Q_h is preferred to Q_{h+1}, $h = 1,\dots,K-1$. To avoid excessive consideration of cases with cost criteria, without loss of generality, hereafter *all criteria are considered as benefit* ones. To form the ordered categories within a multicriteria sorting problem, so called (multicriteria) *limiting* or *central* profiles are assigned (Roy, 1996; Zopounidis & Doumpos, 2002). In this contribution, limiting profiles are considered.

The set of source alternatives, a_i, $i = 1,\dots,n$, and the limiting profiles, P_h, $h = 1,\dots,K_p$, form an *extended set of alternatives* (parameter p depends on a model of decision rule).

As a rule, Limiting Profiles (LPs), P_h, are formed as follows: $P_h = (P_{h1},\dots,P_{hm})$, $h = 1,\dots,K+1$, where FN, P_{hj}, is a LP h for criterion j, and vector of FNs, P_{h+1}, is dominated in Pareto by P_h: $P_{h+1} \prec_{P(M)} P_h$, i.e., $P_{h+1,j} \prec_M P_{hj}$, $j = 1,\dots,m$, (m is the number of criteria for multicriteria sorting problem under consideration, M is a used ranking method).

Based on LPs, the decision rule for assigning an alternative a_i to corresponding category $Q(a_i)$ (Nemery & Lamboray, 2008; Zopounidis & Doumpos, 2002) within TOPSIS-Sort/FTOPSIS-Sort model with ranking method M is as follows:

$$\begin{aligned} &Q(a_i) = Q_h \ if \ D(P_{h+1}) \prec_M D(a_i) \preceq_M D(P_h), \\ &h = 1,\dots,K-1; \\ &Q(a_i) = Q_K \ if \ D(P_{K+1}) \preceq_M D(a_i) \preceq_M D(P_K). \end{aligned} \tag{9}$$

It should be stressed, in fuzzy environment the decision rule (9) needs to be analyzed for correctness. In addition, the following decision rule can also be used (see Section 5.2, Remark 1).

$$\begin{aligned} &Q(a_i) = Q_1 \ if \ D(P_2) \prec_M Q(a_i), \\ &Q(a_i) = Q_h \ if \ D(P_{h+1}) \prec_M D(a_i) \preceq_M D(P_h), \\ &h = 2,\dots,K-1; \\ &Q(a_i) = Q_K \ if \ D(a_i) \preceq_M D(P_K). \end{aligned} \tag{10}$$

For any known ordinary MCDA method with a (benefit) generalized criterion D, the *basic axiom* has the place: if alternative B is dominated in Pareto by alternative A, $B \prec_P A$, then the rank of alternative B cannot be higher of the rank A: $D(B) \le D(A)$ (for MCDA method TOPSIS, if $B \ne A$, the latter inequality is strong, as generalized criterion $D(a)$ (5)/(8) is strictly monotonic for each variable x_{ik}, $i = 1,\dots,n$, $k = 1,\dots,m$). Thus, for ordinary MCDA sorting methods the rule (9) is correct.

However, in fuzzy environment with fuzzy LPs P_h, $h = 1,\dots,K+1$, the basic axiom can be violated in the general case by FMCDA model (Yatsalo, Korobov, & Martínez, 2021; Yatsalo, Korobov, Oztaysi, et al., 2021). The latter can result in the inequality $P_{hj} \prec_M P_{h+1,j}$, and decision rule (9) should be considered as incorrect.

To avoid such a situation, additional requirements must be imposed on LPs. The authors suggest an approach based on the use of *distinguishable* FNs (Yatsalo & Martínez, 2018): FNs $Z_i = \{[A_\alpha^i, B_\alpha^i]\}$, $i = 1,2$, are distinguishable (strictly distinguishable), if one of them, say Z_1, is to the right (strictly to the right) of the another one: $Z_2 \trianglelefteq Z_1$ ($Z_2 \triangleleft Z_1$) iff $A_\alpha^2 \le A_\alpha^1$ *and* $B_\alpha^2 \le B_\alpha^1$ (correspondingly, for the case 'strictly', sign '<' is used), $\alpha \in [0,1]$; the set of FNs $Z_i, i = 1,\dots,n$, are distinguishable (strictly distinguishable), if any two of them are distinguishable (strictly distinguishable).

According to Yatsalo, Korobov, and Martínez (2021), from $Z_2 \trianglelefteq Z_1$ follows $Z_2 \preceq_M Z_1$ ($Z_2 \prec_M Z_1$ for the strictly distinguishable case), $M = CI,\ IM$; moreover, for FTOPSIS models with generalized criterion (5) and proper assessing functions of FNs, $D(Z_2) \trianglelefteq D(Z_1)$, and the later is also true for generalized criterion (8) when using both SFA (including approximate computing based on Tr/TpFNs) and TMs.

Thus, for the set of (strictly) distinguishable vectors of FNs P_h, $P_{h+1} \triangleleft P_h$ (i.e., $P_{h+1,j} \triangleleft P_{hj}, j = 1,\dots,m$), $D(P_{h+1}) \prec_M D(P_h)$, $h = 1,\dots,K$, for ranking method $M = CI, IM$ and models with the generalized criterion (8). It should be emphasized, the use of distinguishable LPs within FMCDA-Sort models is an intuitively understandable approach, which is implemented in fact in applications.

In addition, a careful analysis of the decision rule (9) shows that for correct implementation of this decision rule in the general case, the marginal LPs, P_1 and P_{K+1}, should be considered as vectors with singletons, otherwise, if FN, e.g., P_{11}, is not a singleton, then an alternative $x = (x_1, x_2, \dots, x_m)$ can be formed such that $D(P_1) \prec_M D(x)$

(e.g., $x_j = P_{1j},\ j = 2, \ldots, m$, and x_1 is a singleton, which is close to the right point of the $supp(P_{11})$ and $P_{11} \prec_M x_1$). Taking also into account that marginal LPs play in decision rule (9) only a technical role, the use of singletons as global minimal and maximal marginal points (e.g., in a scale with the global approach to normalization (6), (7), $x_{K+1,j} = 0$, and $x_{1,j} = 1,\ j = 1, \ldots, m$) is effective in applications.

There can be suggested also another effective approach for sorting alternatives to K categories using $K - 1$ limiting profiles, $P_h,\ h = 1, \ldots, K - 1$, as well as the use of so called *central profiles* (Nemery & Lamboray, 2008; Zopounidis & Doumpos, 2002). In this paper, limiting profiles along with the decision rule (9) (and the decision rule (10) when using SFA/approximate models, see below Remark 1) are considered.

Taking into account different FTOPSIS models, indicated in Section 4.1, the following FTOPSIS based sorting models are introduced in this paper: FTTrCI-Sort and FTTpCI-Sort, FTTrIM-Sort and FTTpIM-Sort, FTSCI-Sort, FTSIM-Sort, FTRCI-Sort, and FTRIM-Sort; with these denotations, e.g., model FTTrCI-Sort means that within this sorting model, FTOPSIS model FTTrCI is implemented along with the decision rule (9) or (10) for sorting alternatives. Thus, the introduced FTOPSIS-Sort models differ by FTOPSIS models, which are the ground for corresponding sorting models, described in Section 4.1; the choice of the generalized criterion, (5) or (8), as well as the decision rule, (9) or (10), depends on the scenario under consideration.

Development of a family of FTOPSIS-Sort models represents a generalization of existing approaches to multicriteria sorting in the fuzzy environment based on a chosen MCDA method (TOPSIS in this contribution).

It should also be stressed, in the case, when marginal LPs (in the normalized scale) are $P_1 = (1, \ldots, 1)$ and $P_{K+1} = (0, \ldots, 0)$, which also are considered as ideal and anti-ideal points in FTOPSIS, and the extended set of alternatives comprises the source alternatives along with all LPs, local and global approaches to normalizing criteria values (Section 4.1) coincide. In such a situation, sorting a given set of alternatives by an FTOPSIS-Sort model can be implemented in one iteration, i.e., for one computing the ranks of the extended (initial, $a_i,\ i = 1, \ldots, n$, and additional, $P_h,\ h = 1, \ldots, K + 1$), set of alternatives. Such an approach also makes it possible to say that there is no rank reversal problem (Belton & Stewart, 2002) for these models.

5. Results: The use of FTOPSIS-Sort models in the case study on healthcare supply chain alternative selection problem

The purpose of this section is to explore within a real-world case study on multi-criteria sorting the distinctions in sorting alternatives when using different FTOPSIS-Sort models.

5.1. Description of the case study

Healthcare supply chain refers to the flow of medical supplies, drugs, and equipment from the point of origin to the point of consumption in the healthcare industry. It includes all the activities involved in the planning, sourcing, purchasing, inventory management, and distribution of medical supplies and equipment to healthcare providers such as hospitals, clinics, and pharmacies. The healthcare supply chain plays a crucial role in ensuring the availability and accessibility of medical supplies and equipment to healthcare providers, which is essential for the delivery of quality healthcare services to patients. A well-managed healthcare supply chain can help to reduce costs, improve quality, and increase efficiency in the healthcare industry. Selecting the best healthcare supply chain is important for several reasons. The healthcare supply chain has a direct impact on patient outcomes. An effective and efficient supply chain can ensure that medical supplies and equipment are available when needed, which can help to reduce complications, improve patient safety, and save lives. The healthcare industry is under constant pressure to reduce costs while maintaining quality. An optimized supply chain can help to reduce inventory costs, minimize waste, and improve operational efficiency, which can lead to significant cost savings. An effective healthcare supply chain can improve the efficiency of healthcare operations. This can include reducing the time required to order and receive medical supplies, minimizing stockouts, and reducing the time and effort required to manage inventory. A well-managed healthcare supply chain can help to ensure the quality of medical supplies and equipment. This can include ensuring that products meet regulatory requirements, are properly stored and transported, and are free from defects. Healthcare supply chain alternative selection is an MCDA problem, which involves the evaluation of multiple criteria and for subsequent choice, ranking or sorting alternatives. The decision-making process is complex and involves trade-offs between conflicting criteria, such as cost-effectiveness and quality of medical supplies. The decision is important because healthcare supply chain management is critical to the delivery of quality healthcare services. Inefficient or ineffective supply chain management can lead to shortages of medical supplies, increased costs, and reduced quality of care. Therefore, selecting the most appropriate healthcare supply chain alternatives is crucial to ensure the efficient and effective delivery of healthcare services. There are some studies in the literature that address healthcare supply chain and MCDA together. Some of the selected studies are presented in the following. Assessing healthcare supply chain resilience is crucial to combat the healthcare crisis exacerbated by COVID-19. MCDA method can indicate key characteristics of a resilient supply chain. A case study on medical supplies supply chains using MCDA and unsupervised machine learning showed redundancy, collaboration, and robustness as critical indicators for a resilient supply chain, while design, communication, and risk management become less important during the pandemic (Zamiela et al., 2022). Rehman and Ali (2022) prioritized resilience strategies for healthcare supply chains, using MCDA techniques that consider severe, probable, and long-lasting risks. The fuzzy AHP was used to assign importance weights to criteria, and the fuzzy TOPSIS was used to prioritize risks. Results indicated that Industry 4.0, multiple sourcing, risk awareness, agility, and global diversification are the most significant resilience strategies. The study proposed a novel MCDA-based approach for ranking resilience strategies in healthcare supply chains. During the COVID-19 pandemic, healthcare centers have faced challenges in maintaining a sufficient supply of medical face masks and shields, which are essential in preventing the spread of the virus. Pamucar et al. (2022) proposed a decision-making approach using MACBETH and a new CODAS method to address the supplier selection problem during the pandemic. The approach is implemented under fuzzy rough numbers to account for uncertainty and is demonstrated through a real-life case study for a hospital in Istanbul, Turkey, which identified job creation and occupational health and safety systems as top criteria. Lagana and Colapinto (2022) provided an overview of published articles on the application of MCDA methods in healthcare supply chain management at the strategic, tactical, and operational levels. Their study categorized 139 articles published between 2006 and 2021 and offers insights for academic researchers, practitioners, and governments. The review allowed for the establishment of guidelines for the selection of appropriate methods for healthcare supply chain management and provided support for the management of issues in the healthcare and pharmaceutical sector, which involves human life and conflicts of interest. Biswas (2020) presented a comparative analysis of the supply chain performances of leading healthcare organizations in India using MCDA framework. The framework utilized the PIPRECIA method to derive the weights of the criteria based on experts' opinions and applies three different frameworks for ranking purposes. The results of the analysis revealed that large-cap firms do not necessarily perform well and demonstrate consistency across the three MCDA frameworks used.

Saraji et al. (2023) proposed a new MEREC-TOPSIS method under spherical fuzzy sets to determine the objective importance of the validated challenges and evaluated five pharmaceutical companies'

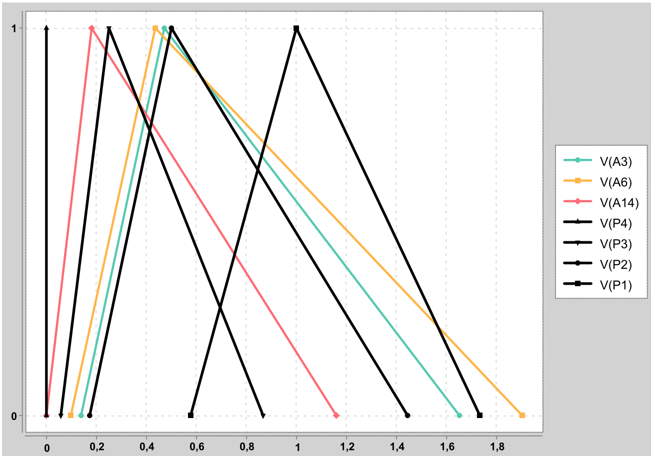

Fig. 1. Generalized criteria of alternatives A_3, A_6, A_{14} and limiting profiles for FTTr[CI,IM]-Sort models by (5).

performance in dealing with the challenges to lean, agile, resilient, and green supply chain adoption. Egala et al. (2023) investigated primary healthcare managers' decision to embrace blockchain technology for primary healthcare by using the best-worst method. Torkayesh et al. (2023) conducted a literature survey of evaluation based on distance from average solution (EDAS) as categorized into nine application groups including healthcare and supply chain management.

However, in the detailed literature review conducted, no study was found that addressed both healthcare supply chain and utilization within this area of multi-criteria sorting methods. This study is the first in the literature whom implement multi-criteria sorting in fuzzy environment to analyze the class of indicated problems.

The objective of this study is to use FMCDA to sort the most appropriate healthcare supply chain options based on 15 alternatives and 4 criteria. These alternatives are: A_1: In-house supply chain management; A_2: Outsourcing to a third-party logistics provider; A_3: Partnership with a local distributor; A_4: Direct shipment from suppliers; A_5: Cross-docking; A_6: Vendor-managed inventory A_7: Collaborative planning, forecasting, and replenishment; A_8: Electronic data interchange; A_9: Radio frequency identification; A_{10}: Just-in-time delivery; A_{11}: Drop-shipping; A_{12}: Pool distribution; A_{13}: Consignment inventory; A_{14}: Forward warehousing; A_{15}: Virtual inventory management.

We identified 4 criteria for the evaluation of the healthcare supply chain alternatives, which are: C_1: Quality (benefit criterion): the quality of the products and services provided, including the accuracy of orders and the reliability of suppliers. C_2: Cost (cost criterion): the total cost of the supply chain, including procurement, transportation, and inventory costs. C_3: Time (cost): The efficiency and timeliness of the healthcare supply chain, including the speed of delivery, lead time, and the time required to manage inventory and order medical supplies. C_4: Sustainability (benefit): The extent to which the supply chain is environmentally and socially sustainable.

For setting criteria values, the linguistic terms set are used, Table 4. The limiting profiles for criterion C_j, $j = 1...,4$, are indicated in Table 5. These four limiting profiles (LPs) define three ordered categories, G_h, $h = 1,2,3$, with the following interpretation: G_1: the category of effective alternatives; G_2: slightly effective; G_3: poorly effective. LPs are interpreted as follows: $\mu_{P_{hj}}(x)$ is a measure of belonging a (real) point x to a LP P_{hj}, and corresponding FN is interpreted as a LP with its inherent uncertainty (Dubois & Prade, 1997).

The performance table (criteria values for all alternatives) is presented in Table 6 with the use of linguistic terms, described in Table 4. It should be stressed that for a cost criterion the big value (e.g., H) means a low (lowest) quality, and this value is transformed at the normalization stage in accordance with Eq. (7).

Table 4
Linguistic terms and their TrFNs correspondences.

Linguistic term	TrFN
Low (L)	(0, 0, 0.25)
Between Low and Middle (LM)	(0, 0.25, 0.5)
Middle (M)	(0.25, 0.5, 0.75)
Between Middle and High (MH)	(0.5, 0.75, 1)
High (H)	(0.75, 1, 1)

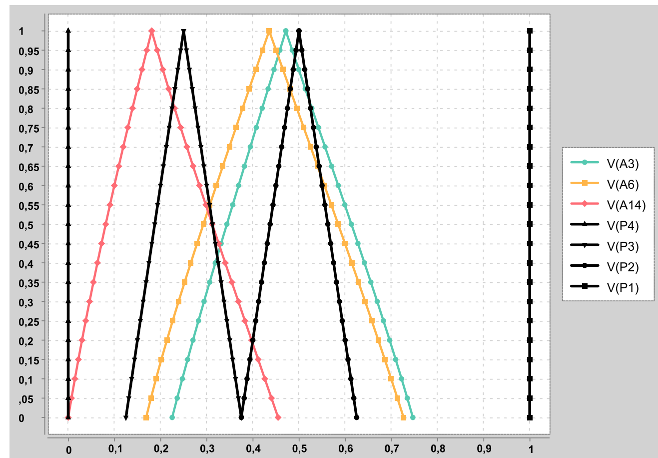

Fig. 2. Generalized criteria of alternatives A_3, A_6, A_{14} and limiting profiles for FTR[CI,IM]-Sort models by (5).

Fuzzy weight coefficients have been evaluated as averaging judgments in linguistic scale by three experts (which analyzed the source information from Samanlioglu and Kaya (2020)), Table 7. The decision rule (9) is used for assigning alternative A_i, $i = 1, \dots, 15$, to one of the categories Q_h, $h = 1,2,3$.

5.2. Comparison and discussion of results on sorting alternatives by different FTOPSIS-Sort models

Based on input values, Tables 5, 6, and 7, alternatives A_i, $i = 1, \dots, 15$, are assigned to categories G_1, G_2, G_3, using FTOPSIS-Sort models, introduced in Section 4.2; wherein, each model is implemented with approaches (5) and (8) for determining the generalized criterion of FTOPSIS-Sort model. The results of sorting alternatives are shown in Tables 8 and 9; the values $I(D(A_i))$, $I = CI$, IM, $i = 1, \dots, 15$, are indicated in brackets. For all computations, the numbers of α-cuts $N_\alpha = 20$ and $N_\alpha = 40$ were used; the output results are the same for both cases. For comparison, the Table 8 also shows the results of sorting alternatives by ordinary MCDA Sorting method - TOPSIS-Sort; mean values of corresponding FNs (Tables 5, 6, 7) are used in this case.

The values $I(D(P_1)), \dots, I(D(P_4))$, $I = CI$, IM, for corresponding models are as follows:

FTTrCI-Sort (5)/(8): 1.103, 0.706, 0.391, 0.0/1.0, 0.5, 0.279, 0.0
FTTrIM-Sort (5)/(8): 1.078, 0.654, 0.356, 0.0/1.0, 0.5, 0.271, 0.0
FTSCI-Sort (5)/(8): 1.075, 0.651, 0.354, 0.0/1.0, 0.5, 0.272, 0.0
FTSIM-Sort (5)/(8): 1.048, 0.596, 0.316, 0.0/1.0, 0.5, 0.265, 0.0
FTRCI-Sort (5)/(8): 1.0, 0.5, 0.25, 0.0/1.0, 0.5, 0.25, 0.0
FTRIM-Sort (5)/(8): 1.0, 0.5, 0.25, 0.0/1.0, 0.5, 0.25, 0.0
TOPSIS-Sort (5): 1.0, 0.5, 0.25, 0.0.

FNs, which are values of generalized criterion for alternatives A_3, A_6, A_{14}, and limiting profiles are presented in Figs. 1 and 2 for approximate and proper models with generalized criterion (5), and in Figs. 3 and 4 for approximate and proper models with generalized criterion (8) correspondingly.

Figs. 1–4 show a significant overestimation of the value of generalized criterion (5), Figs. 1 and 3, in comparison with generalized

Table 5
Limiting profiles.

Criteria	Limiting profiles			
	$P_{4,j}$	$P_{3,j}$	$P_{2,j}$	$P_{1,j}$
C_1: Common satisfaction of the society (Benefit)	0	(0.125, 0.25, 0.375)	(0.375, 0.5, 0.625)	1
C_2: Maximum required time to implement (Cost)	1	(0.625, 0.75, 0.875)	(0.375, 0.5, 0.625)	0
C_3: Financial burden to the economy (Cost)	1	(0.625, 0.75, 0.875)	(0.375, 0.5, 0.625)	0
C_4: Effectiveness on virus spread (Benefit)	0	(0.125, 0.25, 0.375)	(0.375, 0.5, 0.625)	1

Table 6
Performance table.

Alternatives	C_1: Common satisfaction of the society (Benefit)	C_2: Maximum required time to implement (Cost)	C_3: Financial burden to the economy (Cost)	C_4: Effectiveness on virus spread (Benefit)
A_1	LM	LM	MH	H
A_2	L	LM	H	H
A_3	MH	L	LM	MH
A_4	MH	M	MH	MH
A_5	H	MH	M	M
A_6	LM	MH	M	LM
A_7	L	L	L	MH
A_8	L	H	H	H
A_9	L	L	H	M
A_{10}	L	H	H	MH
A_{11}	H	MH	MH	M
A_{12}	H	MH	MH	MH
A_{13}	L	LM	H	H
A_{14}	L	LM	L	LM
A_{15}	L	LM	MH	H

Table 7
Weighting criteria depending on three expert views.

	C_1: Common satisfaction of the society	C_2: Maximum required time to implement	C_3: Financial burden to the economy	C_4: Effectiveness on virus spread
Expert 1	MH (0.5, 0.75,1)	H (0.75, 1, 1)	MH (0.5, 0.75, 1)	H (0.75, 1, 1)
Expert 2	H (0.75, 1, 1)	MH (0.5, 0.75,1)	M (0.25,0.5,0.75)	MH (0.5, 0.75,1)
Expert 3	LM (0, 0.25, 0.5)	M (0.25, 0.5, 0.75)	H (0.75, 1, 1)	H (0.75, 1, 1)
Aggregated Fuzzy Criteria Weights	(0.42, 0.67, 0.83)	(0.5, 0.75, 0.92)	(0.5, 0.75, 0.92)	(0.67, 0.92, 1)

Table 8
Sorting alternatives to categories by FTOPSIS-Sort and TOPSIS-Sort models with generalized criterion (5).

Method	Categories		
	G_3	G_2	G_1
FTTrCI-Sort		A_{14} (0.447), A_7 (0.534), A_9 (0.639)	A_2 (0.739), A_{13} (0.739), A_{15} (0.752), A_3 (0.754), A_6 (0.812), A_1 (0.823), A_8 (0.847), A_{10} (0.877), A_5 (1.072), A_{11} (1.143), A_4 (1.235), A_{12} (1.389)
FTTrIM-Sort		A_{14} (0.38), A_7 (0.488), A_9 (0.59)	A_1 (0.764), A_3 (0.683), A_2 (0.699), A_{13} (0.699), A_{15} (0.701), A_6 (0.718), A_8 (0.805), A_{10} (0.818), A_5 (0.962), A_{11} (1.027), A_4 (1.097), A_{12} (1.237)
FTSCI-Sort		A_{14} (0.377), A_7 (0.488), A_9 (0.586)	A_3 (0.682), A_2 (0.691), A_{13} (0.691), A_{15} (0.695), A_6 (0.715), A_1 (0.755), A_8 (0.799), A_{10} (0.814), A_5 (0.949), A_{11} (1.018), A_4 (1.091), A_{12} (1.216)
FTSIM-Sort	A_{14} (0.298)	A_7 (0.437), A_9 (0.532)	A_3 (0.603), A_6 (0.608), A_{15} (0.64), A_2 (0.648), A_{13} (0.648), A_1 (0.691), A_{10} (0.75), A_8 (0.754), A_5 (0.824), A_{11} (0.888), A_4 (0.932), A_{12} (1.042)
FTRCI-Sort	A_{14} (0.207)	A_7 (0.38), A_6 (0.442), A_9 (0.452), A_3 (0.481)	A_{15} (0.544), A_2 (0.563), A_{13} (0.563), A_1 (0.568), A_5 (0.614), A_{10} (0.634), A_8 (0.662), A_{11} (0.665), A_4 (0.672), A_{12} (0.768)
FTRIM-Sort	A_{14} (0.199)	A_7 (0.371), A_6 (0.44), A_9 (0.449), A_3 (0.479)	A_{15} (0.545), A_2 (0.567), A_{13} (0.567), A_1 (0.573), A_5 (0.619), A_{10} (0.636), A_8 (0.667), A_{11} (0.671), A_4 (0.675), A_{12} (0.772)
TOPSIS-Sort	A_{14} (0.157)	A_7 (0.298), A_6 (0.439), A_3 (0.457), A_9 (0.465)	A_{15} (0.521), A_2 (0.562), A_{13} (0.562), A_1 (0.566), A_{10} (0.618), A_5 (0.641), A_8 (0.643), A_4 (0.69), A_{11} (0.714), A_{12} (0.789)

Table 9
Sorting alternatives to categories by FTOPSIS-Sort models with generalized criterion (8).

Method	Categories		
	G_3	G_2	G_1
FTTrCI-Sort	A_{14} (0.239)	A_7 (0.386), A_6 (0.45), A_9 (0.455), A_3 (0.48)	A_{15} (0.542), A_2 (0.563), A_{13} (0.563), A_1 (0.568), A_5 (0.6), A_{10} (0.62), A_{11} (0.641), A_4 (0.643), A_8 (0.653), A_{12} (0.729)
FTTrIM-Sort	A_{14} (0.225)	A_7 (0.377), A_6 (0.446), A_9 (0.452), A_3 (0.478)	A_{15} (0.544), A_2 (0.567), A_{13} (0.567), A_1 (0.573), A_5 (0.607), A_{10} (0.626), A_{11} (0.651), A_4 (0.653), A_8 (0.659), A_{12} (0.742)
FTSCI-Sort	A_{14} (0.228)	A_7 (0.381), A_6 (0.447), A_9 (0.453), A_3 (0.481)	A_{15} (0.543), A_2 (0.564), A_{13} (0.564), A_1 (0.568), A_5 (0.605), A_{10} (0.627), A_{11} (0.65), A_4 (0.653), A_8 (0.658), A_{12} (0.739)
FTSIM-Sort	A_{14} (0.213)	A_7 (0.372), A_6 (0.444), A_9 (0.45), A_3 (0.478)	A_{15} (0.545), A_2 (0.568), A_{13} (0.568), A_1 (0.573), A_5 (0.612), A_{10} (0.632), A_{11} (0.66), A_4 (0.662), A_8 (0.665), A_{12} (0.752)
FTRCI-Sort	A_{14} (0.207)	A_7 (0.38), A_6 (0.442), A_9 (0.452), A_3 (0.481)	A_{15} (0.544), A_2 (0.563), A_{13} (0.563), A_1 (0.568), A_5 (0.614), A_{10} (0.634), A_8 (0.662), A_{11} (0.665), A_4 (0.672), A_{12} (0.768)
FTRIM-Sort	A_{14} (0.199)	A_7 (0.371), A_6 (0.44), A_9 (0.449), A_3 (0.479)	A_{15} (0.545), A_2 (0.567), A_{13} (0.567), A_1 (0.573), A_5 (0.619), A_{10} (0.636), A_8 (0.667), A_{11} (0.671), A_4 (0.675), A_{12} (0.772)

criterion (8) of proper models, Figs. 2 and 4. At the same time, the overestimation of generalized criterion (5), Fig. 1, noticeable exceeds the overestimation of generalized criterion (8), Fig. 3. It should be stressed that the α-cuts of output FNs for models with approximate computations, FTTr[CI,IM] and FTS[CI,IM], for α=1 and α=0 coincide.

The indicated output FNs, except the crisp marginal ones as well as FNs for approximate models with generalized criterion (5), Fig. 1, are close to symmetric TrFNs, Fig. 2, and the latter is a reason that values $CI(D(P_i))$ and $IM(D(P_i))$, $i = 2, 3$, as well as $CI(D(A_i))$ and $IM(D(A_i))$, $i = 1, \dots, 15$, are close.

Remark 1. Limiting profile P_1, as an additional alternative within the sorting process, dominates in Pareto all alternatives A_i, $i = 1, \dots, 15$, and P_1 and A_i are distinguishable FNs (Yatsalo, Korobov, & Martínez, 2021; Yatsalo & Martínez, 2018) (see also Section 4.2). However, according to Table 8, $I(D(P_1)) < I(D(A_4))$ and $I(D(P_1)) < I(D(A_{12}))$, $I = CI, IM$, for models FTTrCI and FTTrIM; these inequalities have also the place for model FTSCI. The latter inequalities confirm violation of the BA by the indicated models and demonstrates that the hypothetical Proposition 7 in Yatsalo, Korobov, and Martínez (2021) (that utilization of distinguishable criteria values within FMCDA models does not lead to violation od the BA, which was proved for FTOPSIS models with generalized criterion in form (8)) has no place for approximate/SFA FTOPSIS models with generalized criterion in the form (5). The latter is also a ground for (possible, in the general case) violating the property of monotonicity (Nemery & Lamboray, 2008; Zopounidis & Doumpos, 2002) (if alternative B is dominated by alternative A in Pareto, $B \prec_P A$, then category of B cannot exceed category of A) by approximate/SFA FTOPSIS-Sort models, which are based on the use of generalized criterion (5).

Remark 2. In Yatsalo, Radaev, and Martínez (2022), the authors suggested an approach for analysis of distinctions withing ranking alternatives by different models with the use of the *square of linguistic distinctions* based on linguistic variable "Level of Distinction". For this, three zones/terms to analyze distinctions in ranking FNs (with normalized support in [0,1]) based on *differences* of defuzzified values of FNs for a defuzzification based ranking method has been suggested with the following interpretation: Z_1 = [0, 0.01] - *no/negligible* distinctions, Z_2 = (0.01, 0.1] - *noticeable* distinctions, and Z_3 = (0.1,1] - *significant* distinctions. In this contribution, the implementation of such a concept within FMCDA-Sort models is not considered.

Tables 8 and 9 allow analyzing the robustness (at least partly) of assigning an alternative to the specific category based on the family of models under consideration. E.g., alternative A_3 (0.481), Table 9, is assigned by the model FTRCI-Sort to the category G_2; the distance of the value $CI(D(A_3))$ to the value $CI(D(P_2))$ (here P_2 is the limiting profile) is 0.5-0.481=0.019; i.e., alternative A_3 is close to category G_1 (though the distance is higher than the negligible value 0.01, see Remark 2).

According to the results of Table 8, different FTOPSIS-Sort models, which are based on the generalized criterion (5), can result in slightly different sorting alternatives (within this case study, for linguistic terms as input values). At the same time, for the same set of alternatives, there is no difference in sorting alternatives by FTOPSIS-Sort models with the generalized criterion (8), Table 9. Is this an accidental situation specific only for this case study on sorting alternatives, when different FTOPSIS-Sort models with the generalized criterion (5) result in different results on sorting alternatives, while models based on the generalized criterion (8) have the same outcome? To answer such questions, a more general problem can be explored: what is the frequency (statistical assessment of probability) that the use of different FTOPSIS-Sort models, say M_1 and M_2, leads to different results on sorting the same set of alternatives? One of the approaches to exploring this problem is considered in next Section.

The authors consider the proposed family of FTOPSIS-Sort models as an effective tool for multi-criteria sorting of alternatives in the fuzzy environments, which can be implemented for exploring different actual real-world case studies, e.g., environmental protection and energy-efficient resource allocation (Mohajer et al., 2023, 2022).

The use of the family of FMCDA-Sort/FTOPSIS-Sort models has, among others, the following methodological advantages in comparison with utilizing a separate (approximate) model:

- comparison of the output results by approximate and proper models from the family of models under implementation; in the case of absence of significant distinctions, the use of (more simple and cheap) approximate models may be considered as justified for the corresponding class of scenarios;

- the coincidence of the sorting alternatives when using different models of the family, in fact, indicates some (relative) robustness of the output results within the multi-criteria sorting; conversely, notable differences actually indicate that sorting alternatives is not robust in terms of internal and, in the general case, external (Belton & Stewart, 2002) uncertainties;

- with the methodological point of view, the use of the family of FMCDA-Sort/FTOPSIS-Sort models shows the need for careful attention to the choice of a model for applications due to the prevailing present concept of "model adequacy" (Yatsalo, Radaev, & Martínez, 2022) (an author has developed a model and suggests to use it in applications without a deep analysis of its properties).

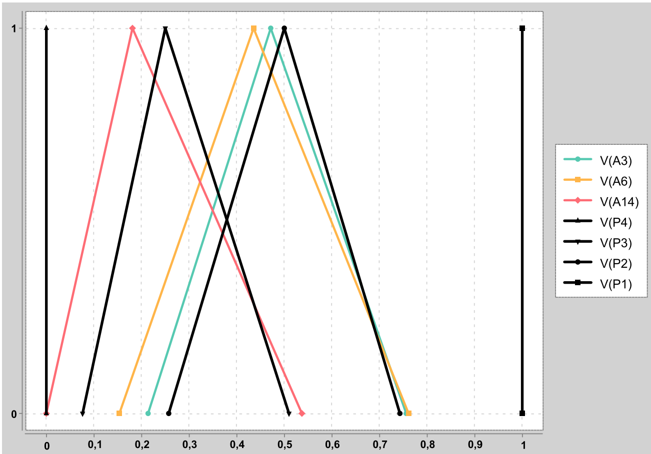

Fig. 3. Generalized criteria of alternatives A_3, A_6, A_{14} and limiting profiles for FTTr[CI,IM]-Sort models by (8).

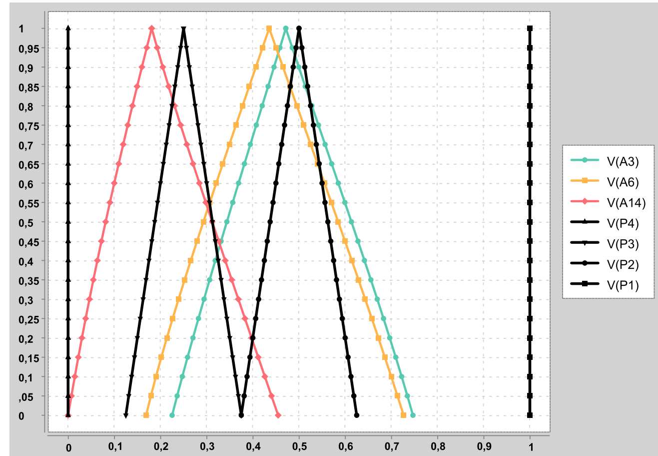

Fig. 4. Generalized criteria of alternatives A_3, A_6, A_{14} and limiting profiles for FTR[CI,IM]-Sort models by (8).

6. Study: Analysis of the distinctions on sorting alternatives by different FTOPSIS-Sort models

In this section, the statistics of distinctions on sorting alternatives by different FTOPSIS-Sort models is explored with the use of Monte Carlo simulating input scenarios. The following FTOPSIS-Sort models are considered: FTTrCI-Sort, FTPpCI-Sort, FTTrIM-Sort, FTTpIM-Sort, FTSCI-Sort, FTSIM-Sort, FTRCI-Sort, and FTRIM-Sort.

6.1. Setting scenarios for distinction analysis

In this subsection, the approach to exploring the distinctions in sorting alternatives by different FTOPSIS-Sort models is described. The suggested approach is based on the implementation of Monte Carlo technology, which consists here in generating random numbers (by the chosen random number generator) with subsequent forming the input scenarios along with corresponding input information.

1. Multi-criteria sorting problems with $m = 5$ criteria and $n = 20$ alternatives with $K = 4$ categories are set. All criteria are considered as *benefit* ones. Scenarios for exploring distinctions in sorting alternatives by different FTOPSIS-Sort models are generated through Monte Carlo simulation of *input* criteria values and weight coefficients. The number of Monte Carlo iterations, N_{it}, is $N = N_{max}$ with intermediate output of the results for number of iterations N_1, $N_1 < N_{max}$. In this contribution, $N_1 = 10000$, $N_{max} = 20000$.
Random number generator (Mersenne Twister (Matsumoto & Nishimura, 1998)) with uniform distribution in segment [0,1], which is utilized in this work, meets the necessary requirements for use within the tasks of this class (its cycle period exceeds $2^{19937} - 1$).

   (a) Within *each iteration*, four types of scenarios are generated:

      - scenario with *symmetric* Trapezoidal FNs (TpFNs);
      - scenario with *symmetric* TrFNs;
      - scenario with the *terms* of a linguistic variable (LTs);
      - scenario with crisp values.

   (b) To generate a *symmetric* TpFN, (a, b, c, d), the following algorithm is used: three points are randomly generated in [0,1]; let us denote these points, in ascending order, as $a \le x \le d$. If $x < e = (a + d)/2$, then we put $b = x$, and the point c is determined as the symmetric one for b concerning the point e; four points, (a, b, c, d), form a symmetric TpFN; if $x > e$, then $c = x$ and the point b is determined similarly to the previous case; it should be added, if $|x - e| \le 0.01$, TpFN (a, b, c, d) is substituted by TrFN (a, e, d);

   (c) Forming a symmetric TrFN, *which is associated with TpFN* (a, b, c, d), is based on the use of TrFN (a, e, d), where indicated points, a, e, d, have been defined in previous item.

   (d) Fuzzy linguistic variable with the standard 7-terms scale (Pedrycz et al., 2011), where each term is represented by TrFN, is considered; a term is generated using discrete uniform distribution with seven points.

   (e) Mean value of any TrFN (a, x, d), including that for the marginal linguistic terms, defined above, is also used in the scenario with crisp values;

   (f) The indicated approaches to generating FNs for corresponding scenarios are implemented for forming the performance table, x_{ij}, with n alternatives and m criteria, as well as for weight coefficients, w_j, $i = 1, \dots, n$, $j = 1, \dots, m$ (see below).

   (g) After random generating n FNs (criteria values), x_{ij}, for criterion j, $j = 1, \dots, m$, the range r_j of values e_{ij}, $i = 1, \dots, n$ (middle points of generated TrFNs/TpFNs), as the distance between minimal and maximal values of mean points for fixed j, is determined. If $r_j \le 0.01$, this sub-iteration is canceled, and a new one on generating criteria values for criterion j is implemented.

   (h) Generating weight coefficients, w_j, $j = 1, \dots, m$, for FTOPSIS-Sort models with Tr/TpFNs input values and linguistic terms is implemented in accordance with the following algorithms.

      i. Implementation of an approach, which is close to F-swing weighting method (Yatsalo et al., 2017): for criterion j_0 with the biggest range among values r_j, $j = 1, \dots, m$, of criterion values (item (g) above), weight coefficient $w_{j_0} = 1$ is assigned. Other weight coefficients are generated randomly in accordance with items (b)/(c) described above and are interpreted as fractions from the most weighted criterion; weight coefficients with Tr/TpFNs are used correspondingly in models with input Tr/TpFNs.

ii. Setting weight coefficients for FTOPSIS-Sort models with linguistic terms as input values is based on simulating "experts' judgments" in accordance with that in Section 5.2, Table 7. For each criterion j, $j = 1, \dots, m$, five terms are simulated (the same 7-term scale is used as for specifying the criterion), then TrFN, w_j, is calculated, which is the average of the generated TrFNs.

2. For (a fixed) criterion j, $j = 1, \dots, m$, Limiting Profiles (LPs), P_{hj}, $h = 1, \dots, K+1$, are set in accordance with the following approaches.

   (a) The first approach is based on a *uniform partition* of the segment [0,1] into K equal (in length) segments using $K+1$ points, p_k, $k = 1, \dots, K+1$; $p_{k+1} < p_k$, and $p_1 = 1$, $p_{K+1} = 0$.
   (b) Within the second approach, $K-1$ points, p_k, $k = 2, \dots, K$, are randomly generated in interval (0,1); as above, points are numerated in descending order, and $p_1 = 1$, $p_{K+1} = 0$. If $p_k - p_{k+1} \leq 0.01$ for at least one k, $k = 1, \dots, K$, this sub-iteration is canceled and new one is generated.
   (c) Forming LPs based on the points p_k, $k = 1, \dots, K+1$, is as follows. Each segment $[p_{k+1}, p_k]$, $k = 1, \dots, K$, is subdivided in s equal sub-segments (in this contribution, $s = 3$); let $d_{k+1,k}$ be the length of this sub-segment (for the first approach, item (a), $d_{k+1,k} = d$, $k = 1, \dots, K$), and $d_k = \min\{d_{k+1,k}, d_{k,k-1}\}$, $k = 2, \dots, K$.
   For each h, $h = 2, \dots, K$, symmetric TrFN $P_{hj} = (p_h - d_h, p_h, p_h + d_h)$ is formed; here j is a criterion under consideration, $j = 1, \dots, m$. For criterion j, LPs are: $P_{1j} = 1$, P_{hj}, $h = 2, \dots, K$, $P_{K+1,j} = 0$. It should be stressed once more, these LPs are interpreted as uncertainty of used LPs (Dubois & Prade, 1997).

3. The following problems are explored through Monte Carlo iterations for determining the distinctions in sorting alternatives by the pair of models M_1 and M_2.

   (a) *Assessing the occasions of distinctions for category Q_k (for fixed k) in sorting alternatives by FTOPSIS-Sort models M_1 and M_2.*
   Let $DIS_k(0; M_1, M_2) = 0$, $k = 1, \dots, K$; if at iteration t, $t = 1, \dots, N_{it}$, $N_{it} = N_1, N_{max}$, the category Q_k, as a result of sorting alternatives by model M_1, differs from that by model M_2, i.e., there exists at least one alternative A that belongs to Q_k when sorting by M_1 (M_2) and does not belong to Q_k according to sorting by M_2 (M_1), then $d_k(t; M_1, M_2) = 1$; if there is no distinction, $d_k(t; M_1, M_2) = 0$; the number of distinctions for t iterations is assessed as

$$DIS_k(t; M_1, M_2) = DIS_k(t-1; M_1, M_2) + d_k(t; M_1, M_2). \quad (11)$$

   The probability (statistical assessment) of distinctions for category Q_k in sorting alternatives by models M_1 and M_2 for N_{it} iterations is evaluated as

$$P_k(N_{it}; M_1, M_2) = DIS_k(N_{it}; M_1, M_2)/N_{it}. \quad (12)$$

   (b) *Assessing the occasions of distinctions in sorting alternatives by FTOPSIS-Sort models M_1 and M_2.*
   The distinction, when sorting alternatives by model M_1 and M_2, has the place at iteration t, $d_0(t; M_1, M_2) = 1$, if the outcome of sorting alternatives differs at least for one category Q_k (i.e., $d_k(t; M_1, M_2) = 1$ at least for one k, $k = 1, \dots, K$), otherwise, $d_0(t; M_1, M_2) = 0$. The number of distinctions on sorting alternatives by models M_1 and M_2 is evaluated as follows: $DIS_0(0, M_1, M_2) = 0$, and for determining the cases of distinctions for t iterations, Eq. (11) for $k = 0$ is implemented. The probability (statistical assessment) of distinctions in sorting alternatives by models M_1 and M_2 is determined by the expression (12) for $k = 0$.
   (c) *Determining the number of distinctions for category Q_k.* Let at iteration t, $S_{kp}(t, M_p)$ be the set of alternatives in category Q_k according to sorting by model M_p, $p = 1, 2$; let also $|S|$ be the number of elements/alternatives in a set S. The number of distinctions for category Q_k in sorting alternatives by models M_1 and M_2 at iteration t is assessed by the expression $ND_k(t; M_1, M_2) = |S_{k1}(t, M_1) \cup S_{k2}(t, M_2) \setminus S_{k1}(t, M_1) \cap S_{k2}(t, M_2)|$.
   The total number of distinction for category Q_k and models M_1 and M_2 for t iterations, $TND_k(t; M_1, M_2)$, $t = 1, \dots, N_{it}$, is determined as

$$TND_k(t; M_1, M_2) = TND_k(t-1; M_1, M_2) + ND_k(t; M_1, M_2), TND_k(0; M_1, M_2) = 0. \quad (13)$$

   For all categories, the total number of distinctions for t iterations is assessed as

$$TND_0(t; M_1, M_2) = \sum_{k=1}^{K} TND_k(t; M_1, M_2). \quad (14)$$

   In this contribution, when assessing the total number of distinctions $TND_0(t; M_1, M_2)$, the simple arithmetical determining the number of distinctions according to (14) is implemented for practical evaluation; the questions of dependence of distinctions for different categories is not explored.
   For category Q_k, $k = 1, \dots, K$, the mean number of distinctions for N_{it} iterations is estimated as

$$MND_k(N_{it}; M_1, M_2) = TND_k(N_{it}; M_1, M_2)/N_{it}. \quad (15)$$

   For all categories, mean number of distinctions for N_{it} iterations is assessed with Eq. (15) for $k = 0$.

4. *Sorting alternatives*. For FTOPSIS-Sort models with ranking methods CI and IM, at each Monte Carlo iteration, sorting alternatives, a_i, $i = 1, \dots, n$, is implemented as follows: after computing the generalized criteria, $D(a_i)$, as well as $D(P_h)$ (5)/(8)) for all $i = 1, \dots, n$, and $h = 1, \dots, K+1$, each alternative, a_i, is assigned to corresponding category, $Q(a_i)$, based on the decision rule (9) (as well as on the rule (10) for approximate models, Remark 1).

6.2. Evaluation and discussion of the distinctions in sorting alternatives by FTOPSIS-Sort models

The following pairs of FTOPSIS-Sort models are compared during implementing Monte Carlo iterations (duplicate part of model name "-Sort" is omitted below and in corresponding Tables):

FTTrCI-FTTrIM, FTTrCI-FTSCI, FTTrIM-FTSIM, FTSCI-FTSIM, FTSCI-FTRCI, FTSIM-FTRIM, FTRCI-FTRIM.

Symmetric TrFNs/TpFNs, linguistic terms and crisp numbers (for TOPSIS-Sort model) are used as input values. Limiting Profiles are set both by uniform partition and randomly generated in segment (close interval) [0,1] depending on the scenario under consideration. The number of Monte Carlo iterations is N_{max}=20000; the number of α-cuts, $N_\alpha = 20$; generalized criterion is calculated by Eq. (5) except Table 15 with generalized criterion (8).

Based on the results presented in Tables 10–14, the following findings can be formulated.

Table 10
Probability (statistical assessments) of distinctions and the mean number of distinctions by indicated pairs of models for category Q_1/Q_0 (all categories) accordingly, scenario with input symmetric TrFNs and randomly generated limiting profiles.

	FTTrCI-FTTrIM	FTTrCI-FTSCI	FTTrIM-FTSIM	FTSCI-FTSIM	FTSCI-FTRCI	FTSIM-FTRIM	FTRCI-FTRIM
P_1/P_0	0.828/0.881	0.836/0.88	0.959/0.972	0.959/0.973	0.999/1.0	0.96/0.999	0.033/0.177
MND_1/MND_0	1.914/5.126	2.026/5.292	3.959/10.504	3.847/10.365	10.029/28.72	6.185/22.873	0.035/0.399

Table 11
Probability of distinctions and the mean number of distinctions by indicated pairs of models for category Q_1/Q_0 accordingly, scenario with input symmetric TrFNs, limiting profiles are set by uniform partition.

	FTTrCI-FTTrIM	FTTrCI-FTSCI	FTTrIM-FTSIM	FTSCI-FTSIM	FTSCI-FTRCI	FTSIM-FTRIM	FTRCI-FTRIM
P_1/P_0	0.872/0.926	0.897/0.939	0.993/0.998	0.991/0.997	0.999/1.0	0.982/1.0	0.004/0.159
MND_1/MND_0	2.007/4.989	2.193/5.289	4.389/10.695	4.203/10.395	8.721/27.647	4.516/21.023	0.004/0.348

Table 12
Probability of distinctions and the mean number of distinctions by indicated pairs of models for category Q_1/Q_0 accordingly, scenario with input linguistic TrFNs and randomly generated limiting profiles.

	FTTrCI-FTTrIM	FTTrCI-FTSCI	FTTrIM-FTSIM	FTSCI-FTSIM	FTSCI-FTRCI	FTSIM-FTRIM	FTRCI-FTRIM
P_1/P_0	0.47/0.798	0.481/0.804	0.437/0.831	0.427/0.826	0.648/0.982	0.494/0.944	0.034/0.128
MND_1/MND_0	0.693/3.455	0.719/3.534	0.658/3.879	0.632/3.799	1.562/10.614	0.913/6.901	0.034/0.272

Table 13
Probability of distinctions and the mean number of distinctions by indicated pairs of models for category Q_1/Q_0 accordingly, scenario with input linguistic TrFNs, limiting profiles are set by uniform partition.

	FTTrCI-FTTrIM	FTTrCI-FTSCI	FTTrIM-FTSIM	FTSCI-FTSIM	FTSCI-FTRCI	FTSIM-FTRIM	FTRCI-FTRIM
P_1/P_0	0.265/0.669	0.27/0.681	0.236/0.725	0.23/0.714	0.454/0.977	0.282/0.901	0.028/0.124
MND_1/MND_0	0.307/2.157	0.314/2.224	0.269/2.5	0.263/2.433	0.605/6.725	0.33/4.331	0.029/0.263

1. First of all, it should be stressed that the probabilities of distinctions (their statistical assessments) when using different FTOPSIS-Sort models with generalized criterion (5) are significant and sufficiently exceed (with the exception for one pair) the value 0.1 both for one category, Q_1, and for all categories, Q_0 (the level 0.1 is discussed in Yatsalo, Radaev, and Martínez (2022) as a level of significant distinctions). At the same time, the exception is the pair FTRCI-FTRIM with proper assessing functions of FNs; for this pair, distinctions for one category, Q_1, is noticeable, Table 10 (between 0.01 and 0.1), and negligible small, Table 11 (less then 0.01) for this pair; distinctions for Q_0 are (weekly) noticeable, and are between 0.12 and 0.18, except the outcomes in Table 14, where for this pair distinctions for both Q_1 and Q_0 are noticeable when using TpFNs as input FNs. The reason of such significant distinctions in sorting alternatives by models with approximate assessing functions of FNs (FTTr[CI,IM] and FTS[CI,IM]) based on generalized criterion (5) is a significant overestimation of the output FNs, see Fig. 1. To the best of our knowledge, all existing in literature FTOPSIS models are based on approximate computations and the use of generalized criterion (5).
2. Probabilities of distinctions between models with generalized criterion (5), which differ only by ranking methods (e.g., FTTrCI-FTTrIM and FTSCI-FTSIM, Tables 10–13, and FTTpCI-FTTpIM, Table 14) are very significant. Distinctions between models with approximate and proper approaches to assessing functions of FNs and the same ranking methods (FTSCI-FTRCI, FTSIM-FTRIM) are also very significant, Tables 10–13.
   It should be emphasized, distinctions between approximate and correct models say a lot more than distinctions between two approximate models.
3. Comparison of the scenarios with random and uniform setting limiting profiles (Tables 10 and 11, 12 and 13) demonstrates that in all cases distinctions are significant, except the pair of models FTRCI-FTRIM, with proper assessing functions of FNs. This result is exploratory in nature: when solving applied problems, experts decide on the setting limiting profiles depending on the specific cases. At the same time, this result shows that the uniform partition for setting limiting profiles does not lead to more "robust sorting" taking into account complex approaches to robustness analysis.
4. The use of linguistic terms as input values, Tables 12 and 13, results in sufficiently less distinctions in comparison with corresponding pairs of approximate models in Tables 10, 11, and 14. For the pair of proper models, FTRCI-FTRIM, the use of linguistic terms does not lead to less distinctions in comparison with corresponding models in Tables 10, 11, and 14. The latter can be caused by the use non-symmetric marginal TrFNs/terms. At the same time, the use of uniform partition in this scenarios results, for pairs of approximate models, in sufficiently less distinctions (up to 1.4–1.8 times).
5. According to Table 14, distinctions for pairs of approximate models with the generalized criterion (5), when using symmetric TpFNs and *associated* TrFNs as input FNs (Section 6.1, item 1), are significant; e.g., for pair of approximate models FTTrCI-FTTpCI, P_1/P_0 are, correspondingly, 0.582/0.741; at the same time, for the pair of proper models, FTR(Tr)IM-FTR(Tp)IM, the distinctions are sufficiently less: 0.056/0.306, i.e. noticeable for category Q_1, $P_1 = 0.56$, and about two times less (though significant) for all categories, Q_0, P_0=0.306. For proper models with different ranking methods, FTR(Tp)CI-FTR(Tp)IM, the distinctions are 0.015/0.075, that is two times less, than for corresponding models with input TrFNs, Table 10. Significant distinctions between pairs of approximate models with the generalized criterion (5) is explained by a big overestimation of the output FNs.
6. The use of generalized criterion (8) leads to the sufficiently less distinctions (in comparison with those for models based on generalized criterion (5)) between pair of approximate models, Table 15. In this case, distinctions between approximate models with different ranking methods as well as with different computation methods, e.g., FTTrCI-FTSCI, FTSCI-FTSIM, is noticeable for category Q_1: P_1 is between 0.03–0.04, and weakly significant for all categories/Q_0: $P_0 \in (0.16, 0.21)$. Most important are distinctions between approximate and proper models:

Table 14
Probability of distinctions and the mean number of distinctions by indicated pairs of models for category Q_1/Q_0 accordingly, scenario with input symmetric TrFNs/TpFNs and randomly generated limiting profiles.

	FTTpCI-FTTpIM	FTTrCI-FTTpCI	FTTrIM-FTTpIM	FTS(Tr)CI-FTS(Tp)CI	FTR(Tr)IM-FTR(Tp)IM	FTR(Tp)CI-FTR(Tp)IM
P_1/P_0	0.867/1.0	0.582/0.741	0.765/0.845	0.779/0.848	0.056/0.306	0.015/0.075
MND_1/MND_0	1.169/2.98	0.704/1.922	1.52/4.209	1.603/4.337	0.064/0.766	0.015/0.158

Table 15
Probability of distinctions and the mean number of distinctions by indicated pairs of models for category Q_1/Q_0 accordingly, based on generalized criterion (8); scenario with input symmetric TrFNs and randomly generated limiting profiles.

	FTTrCI-FTTrIM	FTTrCI-FTSCI	FTTrIM-FTSIM	FTSCI-FTSIM	FTSCI-FTRCI	FTSIM-FTRIM	FTRCI-FTRIM
P_1/P_0	0.04/0.206	0.032/0.175	0.03/0.168	0.038/0.201	0.077/0.364	0.057/0.278	0.033/0.177
MND_1/MND_0	0.043/0.476	0.034/0.393	0.032/0.374	0.042/0.467	0.09/0.964	0.064/0.677	0.035/0.399

Table 16
Probability of distinctions and the mean number of distinctions by indicated models and TOPSIS model for category Q_1/Q_0 accordingly, scenario with input symmetric TrFNs and crisp values for TOPSIS model, and randomly generated limiting profiles.

	FTTrCI	FTSCI	FTRCI	FTRIM
P_1/P_0	0.999/1.0	0.999/1.0	0.088/0.44	0.065/0.338
MND_1/MND_0	12.059/31.381	10.033/28.781	0.107/1.255	0.074/0.868

FTSIM-FTRIM and FTSCI-FTRCI demonstrate larger distinctions in comparison with previous pairs of models, however they are also insignificant/noticeable for value of P_1, between 0.05 and 0.08, and significant, but sufficiently less for, for probability P_0, 0.25 and 0.4. It should be stressed once more that distinctions between proper models, FTRCI-FTRIM, in Table 15 coincide with distinctions between these models in Table 10, as these models are proper (there is no overestimation for the output FNs as dependence of FNs in these models is taken into account by corresponding computation algorithms).

7. For additional analysis, a number of FTOPSIS-Sort models were compared with the TOPSIS-Sort, Table 16; the input values for TOPSIS-Sort model are crisp values in accordance with the scenario described in Section 6.1, item 1(e). Probability of distinction in sorting alternatives by ordinary TOPSIS-Sort and two approximate FTOPSIS-Sort models (based on generalized criterion (5)), FTTrCI and FTSCI, is practically 1 both for one category, Q_1, and for all categories, Q_0, and mean values of distinctions, MND_1 and MND_0, exceed 10. At the same time, distinctions between TOPSIS-Sort and two proper FTOPSIS-Sort models, FTRCI and FTRIM, are relatively small: insignificant/noticeable for category Q_1 and significant for all categories, Q_0. It can be stressed that the latter distinctions are close, although slightly exceed, then distinctions between proper models and approximate models, FTSCI and FTSIM, with generalized criterion (8), Table 15. Thus, if the researcher does not have proper models (FTRCI or FTRIM), then it is better to use in applications the ordinary sorting model, TOPSIS-Sort, than to sort alternatives based on approximate models, FTTrIM and FTTrCI, with the generalized criterion (5).

The evaluation of the precision of output results is discussed below.

- In Table 17, the output results for the intermediate number of iteration, $N_1 = 10000$, are indicated. Comparison of Table 17 with the corresponding results in Table 10 (with the number of iteration, $N_{max} = 20000$), indicates no significant differences (all differences are less 0.1).
- For evaluating the influence of the α-cut number when determining functions of FNs (including ranking FNs) on the output results, comparison of some models (with different approaches to assessing functions of FNs and different ranking methods) has been implemented. However, comparison of the output results for corresponding models in Tables 18 (the number of α-cuts $N_\alpha = 40$) and 10 ($N_\alpha = 20$) shows no significant differences (differences for all corresponding values are less 0.1).

Thus, the authors state that the output results are valid for inferences about the significance of distinctions when using different FTOPSIS-Sort models.

In accordance with the results presented in this subsection, the smallest distinctions are between the pair of proper models FTRCI-FTRIM, regardless of the chosen scenario, along with the significant distinctions of these models when comparing with approximate ones, FTTr(CI,IM) and FTS(CI,IM). The use of generalized criterion in form (8) leads to less distinctions between pairs of FTOPSIS-Sort models under consideration, Table 15. The indicated results are extremely important when choosing the FTOPSIS-Sort model(s) for application.

7. Conclusions

Multicriteria sorting alternatives is an effective approach especially in the case of a big number of alternatives in the fuzzy environment. In this paper, a family of Fuzzy TOPSIS Sorting (FTOPSIS-Sort) models is introduced based on different approaches to fuzzy extension of ordinary TOPSIS sorting method. The developed models differ by approaches to assessing functions of Fuzzy Numbers (FNs) and methods for ranking of FNs. The suggested approach to forming the family of FMCDA sorting models and in particular FTOPSIS-Sort models is new and has no analogues in the literature.

Despite the contributions and advancements made in the field Fuzzy MCDA (FMCDA) and fuzzy multi-criteria sorting (FMCDA-Sort) models, several limitations should be acknowledged. Firstly, the direct implementation of the extension principle for assessing functions of fuzzy quantities (Fuzzy Numbers, FNs) is ineffective, even for simple functions of FNs. This limitation arises due to the complexity of determining functions of FNs, requiring the use of approximate assessing functions based on basic types of FNs such as triangular FNs (TrFNs) and trapezoidal FNs (TpFNs) or more complicated methods, such as Standard Fuzzy Arithmetic (SFA) and Transformation Methods (TMs) that involve numerical computations along with more complex mathematical algorithms and increased computational time/cost. Secondly, the available fuzzy ranking methods within FMCDA models are numerous, making it challenging to determine the most suitable method for a given scenario. While the study focuses on the Centroid Index (CI) and Integral of Means (IM) ranking methods, there are other methods with different characteristics that could potentially yield different results. This variability in ranking methods adds complexity to the comparison and selection process of FMCDA-Sort models. Additionally, the application of FMCDA-Sort models, including the proposed FTOPSIS-Sort ones, is mainly concentrated in specific areas such as risk assessment,

Table 17
Probability of distinctions and the mean number of distinctions by indicated pairs of models for category Q_1/Q_0 accordingly, scenario with input symmetric TrFNs and randomly generated limiting profiles; $N_1 = 10000$ iteration.

	FTTrCI-FTTrIM	FTTrCI-FTSCI	FTTrIM-FTSIM	FTSCI-FTSIM	FTSCI-FTRCI	FTSIM-FTRIM	FTRCI-FTRIM
P_1/P_0	0.823/0.875	0.832/0.877	0.957/0.969	0.952/0.971	0.995/1.0	0.94/0.999	0.03/0.168
MND_1/MND_0	1.913/5.123	2.015/5.268	3.949/10.428	3.846/10.305	10.018/28.69	6.185/22.873	0.032/0.378

Table 18
Probability of distinctions and the mean number of distinctions by indicated pairs of models for category Q_1/Q_0 accordingly, scenario with input symmetric TrFNs and randomly generated limiting profiles with the number of α-cuts $N_\alpha = 40$.

	FTTrCI-FTTrIM	FTTrCI-FTSCI	FTTrIM-FTSIM	FTSCI-FTSIM	FTSCI-FTRCI	FTSIM-FTRIM	FTRCI-FTRIM
P_1/P_0	0.828/0.88	0.836/0.88	0.957/0.969	0.957/0.969	0.999/1.0	0.96/0.999	0.033/0.176
MND_1/MND_0	1.916/5.107	2.029/5.28	3.952/10.441	3.839/10.297	10.049/28.678	6.213/22.864	0.036/0.4

performance analysis, project selection, and supplier selection. The generalizability and applicability of these models to other domains or industries may require further investigation and validation. Moreover, the comparison and distinction analysis of FMCDA-Sort models, especially those that are fuzzy extensions of the same MCDA method, are still limited in the literature (and are represented at the moment only by the authors' contributions). Furthermore, the case study conducted in the healthcare supply chain domain serves as an illustration of the proposed FTOPSIS-Sort models. However, the findings and conclusions drawn from this specific case study may not be directly applicable to other contexts or industries. Further empirical research and case studies are needed to explore the effectiveness and robustness of the FTOPSIS-Sort models across different domains. Lastly, the study mainly focuses on ordinary fuzzy sets, and there is a lack of exploration of other types of fuzzy sets such as interval type-2 fuzzy sets, intuitionistic, hesitant, and other fuzzy sets within the FMCDA-Sort models. Investigating the potential benefits and limitations of these alternative types of fuzzy sets could enhance the understanding and applicability of FMCDA-Sort models. Additionally, while the proposed FTOPSIS-Sort models offer novel approaches to fuzzy multicriteria sorting and demonstrate promising results in the healthcare supply chain case study, it is important to recognize the limitations inherent in the implementation and generalization of these models.

Concerning application of FTOPSIS-Sort models, the authors can recommend the following. If researchers do not have any proper model and use only the standard generalized criterion (5), the authors recommend sorting alternatives based on ordinary TOPSIS-Sort method with crisp values due to the significant overestimation of the output values. In addition, the use of linguistic variables for criteria values leads to less distinctions (however, they can be still significant), Tables 12 and 13. At the same time, FTOPSIS-Sort models with generalized criterion (8) (both approximate and proper ones) demonstrate sufficiently less distinctions, Table 15.

Subsequent in-depth analysis with the use of Monte Carlo technology for simulating a large number of input scenarios showed that a robust character of sorting alternatives by FTOPSIS-Sort models, which was demonstrated within the case study, is not a rule. It was demonstrated that the probability (evaluated statistically through Monte Carlo iterations) of distinctions in sorting alternatives by any two approximate models with the generalized criterion (5) is very significant, including extremely high distinctions of output results when comparing approximate and proper models (Section 6.2).

According to evaluating distinctions for different FTOPSIS-Sort models with the same approaches to assessing functions of FNs, both approximate and proper ones, the choice of a fuzzy ranking method has also an influence for the output results. The most popular fuzzy ranking method in applications is Centroid Index, CI (Center of Gravity), implemented also in this contribution. However, taking into account the representation of CI through the use of α-cuts (Yatsalo, Korobov, & Martínez, 2021; Yatsalo, Korobov, et al., 2022), the authors state that CI has no advantage over IM (Integral of Means), rather the opposite. According these comments, for sorting alternatives in the fuzzy environment, the authors argue using in applications proper FTOPSIS-Sort models with ranking method IM.

Thus, this paper raises the following problem concerning FMCDA sorting models, which is associated with "presumption of model adequacy in FMCDA" (Yatsalo, Radaev, & Martínez, 2022) and is important from theoretical, methodological, and applied points of view: which of the fuzzy multicriteria sorting model(s) can be recommended for applications?

The analysis of the following problems forms further directions of R&Ds:

- development of the general approach to creating FMCDA sorting models, including different approaches to computing functions of FNs, fuzzy ranking, and an extended set of decision rules for sorting alternatives along with different approaches to setting profiles;
- creating and exploring families of FMCDA-Sort models based on other ordinary MCDA methods (e.g., MAVT, PROMETHEE, etc.);
- exploring the statistics of distinctions taking into account the *level/significance* of distinctions in sorting alternatives by different FMCDA-Sort models based on granulating the output information with the use of linguistic approaches;
- exploring the influence of the dimension of FMCDA sorting problems (the numbers of criteria and categories) on the statistics of distinctions for different FMCDA-Sort models;
- development and exploring families of FMCD-Sorting models for other types of fuzzy sets;
- elaboration of the output information of FMCDA-Sort models for subsequent methodological recommendations.

This work is a part of the R&Ds by the authors on analysis of the features and exploring methodological problems of FMCDA models on ranking and sorting alternatives.

CRediT authorship contribution statement

Boris Yatsalo: Idea of the project, Methodology, Model development, Project administration, Validation, Analysis of the results, Writing paper. **Alexander Radaev:** Computer modules building, Computations, Validation, Writing draft of corresponding sections. **Elif Haktanir:** Survey preparing, Case study building, Writing draft of corresponding sections. **Andrzej M.J. Skulimowski:** Theoretical support, Validation, Editing. **Cengiz Kahraman:** Project administration, Case study building, Theoretical support, Editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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