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Analytical solutions and conservation laws of the generalized model for propagation pulses with four powers of nonlinearity.

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Abstract

Analytical solutions of the generalized nonlinear Schrödinger are presented. Optical solitons corresponding to the mathematical model are given. Conservation laws of the generalized model for propagation pulses with four powers of nonlinearity are considered. The equation studied is the generalization of several well-known models, which allows us to evaluate the influence of various processes on pulse propagation. Conservative quantities for the bright optical soliton are calculated.

Keywords: Generalized model; Conservation law; Optical soliton; Conservative quantity.

1 Introduction

In this paper we study the equation in the form (Kudryashov 2019)

$$i\,q_t + a\,q_{xx} + \left(b_1\,|q|^{2n} + b_2\,|q|^{n} + b_3\,|q|^{-n} + b_4\,|q|^{-2n}\right)\,q = 0, \tag{1}$$

where $q(x,t)$ is a complex function, $i^2 = -1$, $n \in \mathbb{Z}$, a, b_1, b_2, b_3 and b_4 are parameters of Eq. (1), $n \neq -1$ (if $b_1 \neq 0$), $n \neq -2$ (if $b_2 \neq 0$), $n \neq 2$ (if $b_3 \neq 0$), $n \neq 1$ (if $b_4 \neq 0$).

Eq. (1) has been proposed by Kudryashov in 2019 and it is known as the Kudryashov model (Biswas et al 2020d, Raheel et al 2023, Sonmezoglu et al 2022, Yıldırım et al 2020, Zayed et al 2020a,c). Eq. (1) does not pass the Painlevé test (Kudryashov 2019) and the Cauchy problem for this equation cannot be solved by the inverse scattering transform. However, taking into account the traveling wave reduction one can find periodic and solitary wave solutions of Eq. (1) in the form of optical solitons, which have recently been found in a number of papers (see, for example, Arnous et al 2021, Arshed and Arif 2020, Arshed et al 2021, 2022, Biswas et al 2019, 2020a,b,e, Hu and Yin 2022, Kai and Li 2023, Kudryashov 2020b,d,f, 2021d,e, Kudryashov and Antonova 2020, Kumar et al 2020, Li and Wang 2022, Raheel et al 2022a, Raza et al 2021, Zayed and Alngar 2021, Zayed et al 2020d, 2021b,e). Without stopping at a detailed analysis of optical solitons, which are solutions of Eq. (1), we note that Eq. (1) generalizes a number of well-known equations used to describe impulses in optical media. It is obvious that Eq. (1) at $n = 1$, $b_2 = 0$, $b_3 = 0$ and $b_4 = 0$ is the famous non-linear Schrödinger equation, which was the first mathematical model that was proposed to describe the optical solitons (Hasegawa and Tappert 1973a,b, Tai et al 1986).

Thus, optical solitons described by Eq. (1) are currently well studied. However, to the best of our knowledge, conservation laws corresponding to Eq. (1) have not been studied to date. The purpose of this paper is to present conservation laws of Eq. (1) using direct calculations.

It is well known that we say there exists the conservation law corresponding to Eq. (1), if we can write this equation in the form

$$\frac{\partial T_j}{\partial t} + \frac{\partial X_j}{\partial x} = 0, \qquad (j = 1,\, 2,\, 3), \tag{2}$$

where $T \equiv T_j(u, u_x, u_t, \ldots, x, t)$ is the density and $X_j \equiv X(u, u_x, u_t, \ldots, x, t)$ is the flux.

Integrating Eq. (2) with respect to x, we get the conservative quantity of the density as follows (Alshehri et al 2022a, Alshehri and Biswas 2022, Alshehri et al 2022b, Arnous et al 2022, Biswas et al 2020c, 2021a,b, Kivshar and Agrawal 2003, Kivshar and Malomed 1989, Kivshar and Pelinovsky 2000, Kudryashov et al 2022, Olver 1993, Serkin and Belyaeva 2018, Vega-Guzman et al 2021, Yıldırım et al 2021, Zayed et al 2021a, 2020b, 2021c,d,f)

$$I_j = \int_{-\infty}^{\infty} T_j \, dx = Constant. \tag{3}$$

One can see that I_j is the conservative quantity for the solution $q(x,t)$.

This paper is organized as follows. Periodic and solitary wave solutions of Eq. (1) are given in Section 2. Bifurcations of phase portraits of the traveling wave reduction of Eq.(1) are presented in Section 3. In Sections 4, 5 and 6 we obtain conservation laws corresponding to Eq. (1). In Section 75 conservative quantities of optical soliton of Eq. (1) are calculated.

2 Optical solitons of equation

The Cauchy problem for Eq. (1) cannot be soled by the inverse scattering transform (Kudryashov 2019). However the optical soliton of Eq. (1) can be found using the traveling wave solutions. These solutions can be found using special methods (see, for example, Alotaibi 2021, Biswas et al 2021c, 2022, Ege 2022, Ekici 2022, Eldidamony et al 2022a,b, González-Gaxiola 2022, Kudryashov 1991, 1990, 2005, 2009, 2012, 2020a,c,e, 2021a,b,c, 2022a,b,c, Ozisik et al 2022, Raheel et al 2022b, Vitanov 2010, 2011a,b, Vitanov and Dimitrova 2010, Vitanov et al 2010, Wang 2022a,b, Zayed et al 2022).

We take into account traveling wave solutions in the form

$$q(x,t) = y(z)\, e^{i\,(\psi(z)-\omega\, t)}, \tag{4}$$

where $y(z)$, $\psi(z)$ are new functions and $z = x - C_0\, t$.

Substituting (4) into Eq. (1), we obtain the system of equations for imaginary and real part as the following (Kudryashov 2019)

$$2\, a\, y_z\, \psi_z + a\, y\, \psi_{zz} - C_0\, y_z = 0 \tag{5}$$

and

$$\begin{gathered}\omega\, y + C_0\, \psi_z\, y + a\, y_{zz} - a\, y\, \psi_z^2 + \\ b_1\, y^{2\,n+1} + b_2\, y^{n+1} + b_3\, y^{1-n} + b_4\, y^{1-2\,n} = 0.\end{gathered} \tag{6}$$

Eq. (5) can be integrated after being multiplyed by $y(z)$. We have

$$\psi_z = \frac{C_0}{2\,a} - \frac{C_1}{a\, y^2}, \tag{7}$$

where C_1 is an arbitrary constant.

Substituting ψ_z into (6), we get the second-order differential equation for $y(z)$ in the form

$$\begin{gathered}a\, y_{zz} + \left(\omega + \frac{C_0^2}{4\,a}\right) y - \frac{C_1^2}{a\, y^3} + b_1\, y^{2\,n+1} + b_2\, y^{n+1} + \\ b_3\, y^{1-n} + b_4\, y^{1-2\,n} = 0\end{gathered} \tag{8}$$

Multiplying Eq. (8) by y_z and integrating the resulting expression with respect to z, we have at $n \neq -1$, $n \neq -2$, $n \neq 1$ and $n \neq 2$ the first integral as follows

(Kudryashov 2019)

$$y_z^2+\left(\frac{\omega}{a}+\frac{C_0^2}{4\,a^2}\right)y^2+\frac{b_1}{a\,(n+1)}\,y^{2\,n+2}+\frac{2\,b_2}{a\,(n+2)}\,y^{n+2}-$$
$$\frac{2\,b_3}{a\,(n-2)}\,y^{2-n}-\frac{b_4}{a\,(n-1)}\,y^{2-2\,n}+\frac{C_1^2}{a^2\,y^2}=C_2. \tag{9}$$

Some partial cases of Eq. (9) were considered in the paper by Kudryashov in 2019. Here, let us consider Eq.(9) at $C_1=0$ and $C_2=0$ using the new variable in the form (Kudryashov 2019)

$$y(z)=V(z)^{\frac{1}{n}} \tag{10}$$

Substituting the expression (4) into Eq. (9), we have the equation

$$V_z^2-\delta+\alpha\,V-\mu\,V^2+\beta\,V^3-\nu\,V^4=0, \tag{11}$$

where μ, β, ν, α and δ are determined as follows

$$\alpha=\frac{2\,b_3\,n^2}{a(2-n)},\quad \beta=\frac{2\,b_2\,n^2}{a\,(n+2)},\quad \mu=-\frac{\omega\,n^2}{a}-\frac{C_0^2\,n^2}{4\,a^2},$$
$$\nu=-\frac{b_1\,n^2}{a(n+1)},\quad \delta=\frac{b_4\,n^2}{a(n-1)}. \tag{12}$$

The general solution of Eq. (11) is expressed via the Jacobi elliptic sine in the form (Kudryashov 2020b,d,f, 2021d,e)

$$V(z)=V_3+\frac{(V_4-V_3)(V_3-V_1)}{V_3-V_1+(V_1-V_4)\,\mathrm{sn}^2\left\{\sqrt{\chi}(z-z_1);S\right\}}, \tag{13}$$

where

$$\chi=\frac{\nu}{4}\,(V_3-V_1)(V_4-V_2),\quad S^2=\frac{(V_1-V_4)(V_2-V_3)}{(V_2-V_4)(V_1-V_3)} \tag{14}$$

and V_1, V_2, V_3 and V_4 are roots of the following algebraic equation

$$\nu\,V^4-\beta\,V^3+\mu\,V^2-\alpha\,V+\delta=0. \tag{15}$$

These real roots satisfy the following constraints

$$V_1\,V_2\,V_3\,V_4=\frac{\delta}{\nu}, \tag{16}$$

$$V_1\,V_2\,V_3+V_1\,V_2\,V_4+V_1\,V_3\,V_4+V_2\,V_3\,V_4=\frac{\alpha}{\nu} \tag{17}$$

$$V_1\,V_2+V_1\,V_3+V_1\,V_4+V_2\,V_3+V_2\,V_4+V_3\,V_4=\frac{\mu}{\nu}, \tag{18}$$

$$V_1+V_2+V_3+V_4=\frac{\beta}{\nu}. \tag{19}$$

Taking into account (4), (10) and (13), we get the periodic solution of Eq. (1) in the form

$$q(x,t)=\left[V_3+\frac{(V_4-V_3)(V_3-V_1)}{V_3-V_1+(V_1-V_4)\,\mathrm{sn}^2\big\{\sqrt{\chi}(x-C_0t-z_1);S\big\}}\right]^{\frac{1}{n}}\,e^{i\,(\psi(z)-\omega\,t)}, \tag{20}$$

In the case of $V_1=V_2$, we have $S^2=1$ and the elliptic sine is reduced to the hyperbolic tangent. From the solution (13) we obtain the solitary wave solution in the form

$$V(z)=V_3+\frac{(V_4-V_3)(V_3-V_1)}{V_3-V_1+(V_1-V_4)\tanh^2\big\{\sqrt{\chi}(z-z_1)\big\}}. \tag{21}$$

Using (4), (10) and (21), we have the bright and dark soliton of Eq. (1) as follows

$$q(x,t)=\left[V_3+\frac{(V_4-V_3)(V_3-V_1)}{V_3-V_1+(V_1-V_4)\tanh^2\big\{\sqrt{\chi}(x-C_0t-z_1)\big\}}\right]^{\frac{1}{n}}\,e^{i\,(\psi(z)-\omega\,t)}. \tag{22}$$

In particular, the optical soliton of Eq. (1) can be found by looking for the solution of Eq. (9) at $b_3=b_4=C_1=C_2=0$ in the form (see Kudryashov 2020c,e, 2021c, 2022a)

$$q(x,t)=\left[\frac{4\,\mu}{2\,\beta+(\beta^2-4\,\mu\,\nu)\,\mathrm{e}^{-\sqrt{\mu}(x-C_0t-z_0)}+\mathrm{e}^{\sqrt{\mu}(x-C_0t-z_0)}}\right]^{\frac{1}{n}}\,e^{i\,(\psi(z)-\omega\,t)}, \tag{23}$$

where z_0 is a constant and parameters μ, β and ν are determined by (12). Some other optical solitons have been obtained in the paper by Kudryashov in 2019.

3 Bifurcations of phase portraits of the system (8)

In this section we plot several partial cases of phase portraits of the studied system (8). In rewrite it as

$$y_z=v,\ \ v_z=-\frac{b_1}{a}y^{2n+1}-\frac{b_2}{a}y^{n+1}-\left(\omega+\frac{C_0^2}{4a}\right)y-\frac{C_1}{a}\frac{1}{y^3}-\frac{b_3}{a}\frac{1}{y^{n+1}}-\frac{b_4}{a}\frac{1}{y^{2n+1}}. \tag{24}$$

Let us observe the first partial case. We write the equation explored at $n = 1$, $C_1 = 0$ and $b_4 = 0$

$$y_z = v, \;\; v_z = -\frac{b_1}{a} y^3 - \frac{b_2}{a} y^2 - \left(\frac{\omega}{a} + \frac{C_0^2}{4a} \right) y - \frac{b_3}{a} \equiv f_1(y). \tag{25}$$

The first integral of the system (25) is written in a following way

$$H_1(y, v) = \frac{v^2}{2} + \frac{b_1}{a} \frac{y^4}{4} + \frac{b_2}{a} \frac{y^3}{3} + \left(\frac{\omega}{a} + \frac{C_0^2}{4a} \right) \frac{y^2}{2} + \frac{b_3}{a} y. \tag{26}$$

Equilibrium points of Eq. (25) are located on the v axis and with the coordinate defined by the following cubic equation

$$\frac{b_1}{a} y^3 + \frac{b_2}{a} y^2 + \left(\frac{\omega}{a} + \frac{C_0^2}{4a} \right) y + \frac{b_3}{a} \equiv -f_1(y) = 0 \tag{27}$$

Eq. (27) may have at most three roots. Let us denote them as $y_{1s}, \; y_{2s}, \; y_{3s}$. Provided that the root y_{is} is real, the stability of an equilibrium point $(y_{is}, 0)$ is determined by the eigenvalues of the following matrix

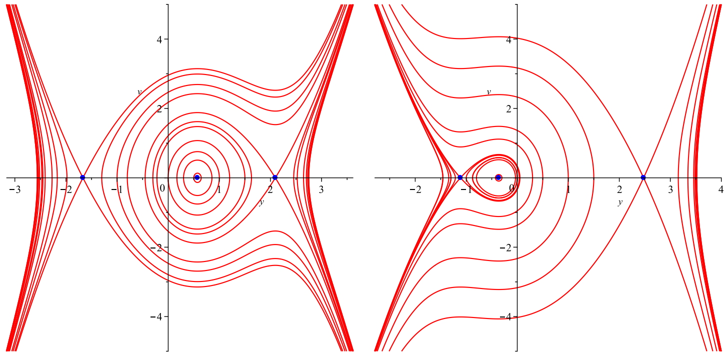
$$\begin{pmatrix} 0 & 1 \\ -\frac{df_1}{dy}\Big|_{y_{is}} & 0 \end{pmatrix}, \;\; i = 1..3. \tag{28}$$

The eigenvalues of (28) are as follows

$$\lambda_{1,2} = \pm \sqrt{\frac{df}{dy}\Big|_{y_{is}}}, \;\; i = 1..3. \tag{29}$$

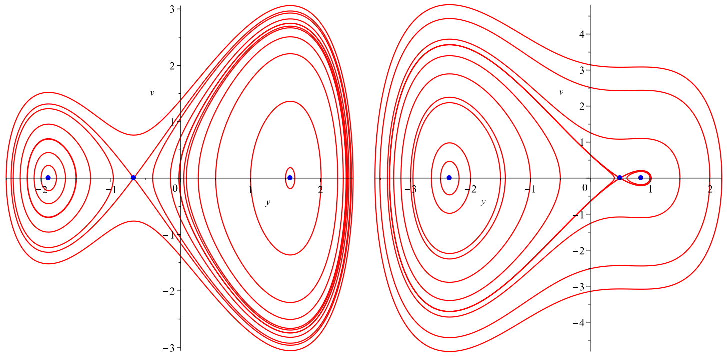
Thus, if $f_1(y)$ increases at the point y_{is}, then an equilibrium is a saddle, if it decreases at that point, then it is a center, otherwise it is a degenerate equilibrium. Having said that, we can propose the following classification of equilibria, depending on the parameter values and the sign of the discriminant D_1 of Eq. (27):

1. $D_1 > 0, \;\; -b_1/a > 0$. Eq. (25) has three equilibria on the v axis with coordinates $y_{1s}, \; y_{2s}, \; y_{3s}$ (we assume $y_{1s} < y_{2s} < y_{3s}$), out of which left and right equilibria are saddles and the middle one is a center (Fig. 1a-1b).
2. $D_1 > 0, \;\; -b_1/a < 0$. Eq. (25) has three equilibria on the v axis with coordinates $y_{1s}, \; y_{2s}, \; y_{3s}$ (we assume $y_{1s} < y_{2s} < y_{3s}$), out of which left and right equilibria are centers and the middle one is a saddle.
3. $D_1 < 0, \;\; -b_1/a > 0$. Eq. (25) has one equilibrium on the v axis with the coordinate y_{1s}, which is a saddle.
4. $D_1 < 0, \;\; -b_1/a < 0$. Eq. (25) has one equilibrium on the v axis with the coordinate y_{1s}, which is a center.



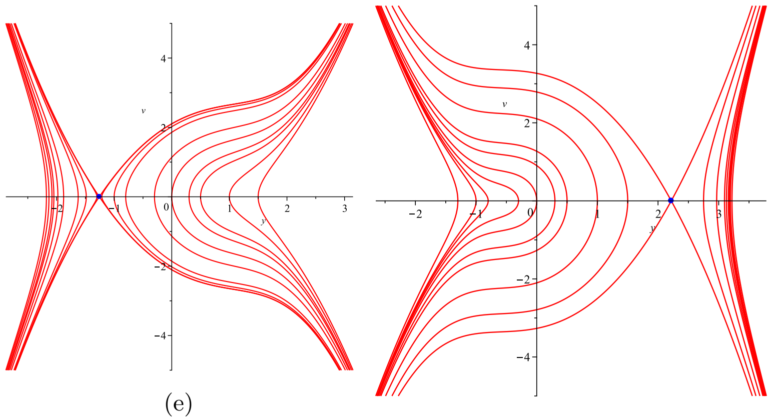
(a) $b_1 = -1,\ a = 1,\ b_2 = 1,\ C_0 = 1,\ b_3 = -2,\ \omega = 3$

(b) $b_1 = -1,\ a = 1,\ b_2 = 1,\ C_0 = 1,\ b_3 = 1,\ \omega = 3$



(c) $b_1 = 1,\ a = 1,\ b_2 = 1,\ C_0 = 1,\ b_3 = -2,\ \omega = -3$

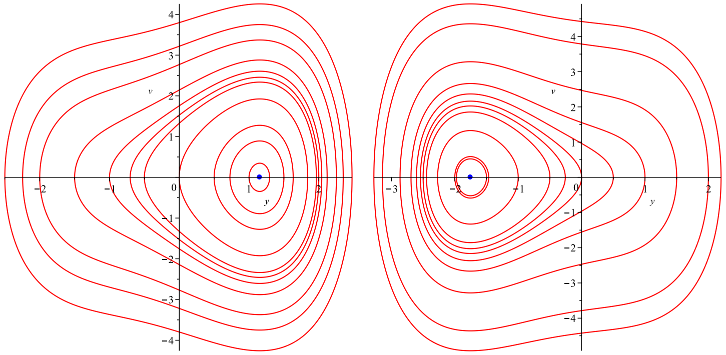
(d) $b_1 = 1,\ a = 1,\ b_2 = 1,\ C_0 = 1,\ b_3 = 1,\ \omega = -3$



(e) $b_1 = -1,\ a = 1,\ b_2 = 1,\ C_0 = 1,\ b_3 = -2,\ \omega = 1$

(f) $b_1 = -1,\ a = 1,\ b_2 = 1,\ C_0 = 1,\ b_3 = 1,\ \omega = 2$

Fig. 1: Phase portraits for the first case

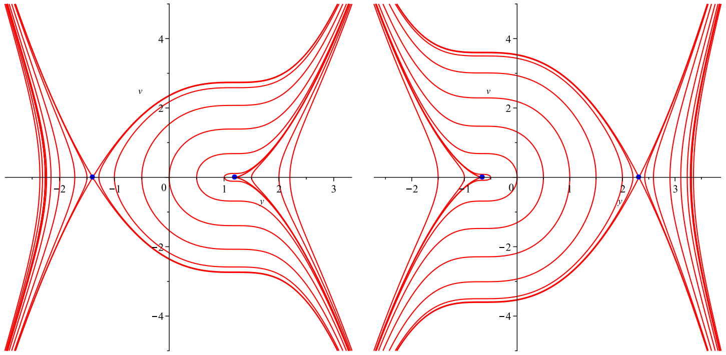


(a)

$b_1 = 1,\ a = 1,\ b_2 = 1,\ C_0 = 1,\ b_3 = -2,\ \omega = -1$

(b)

$b_1 = 1,\ a = 1,\ b_2 = 1,\ C_0 = 1,\ b_3 = 1,\ \omega = -1$

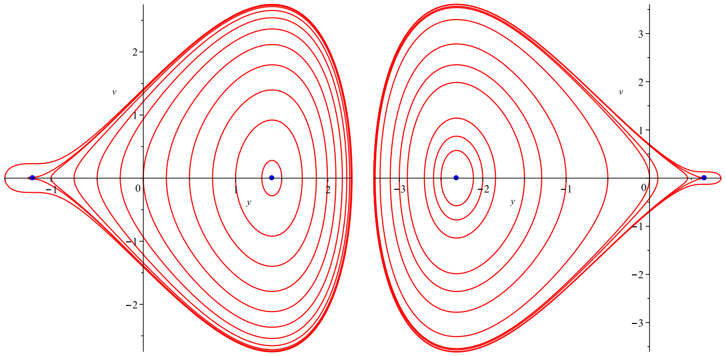


(c)

$b_1 = -1,\ a = 1,\ b_2 = 1,\ C_0 = 1,\ b_3 = -2,\ \omega \approx 1.656$

(d)

$b_1 = 1,\ a = 1,\ b_2 = 1,\ C_0 = 1,\ b_3 = 1,\ \omega \approx 2.36$



(e)

$b_1 = 1,\ a = 1,\ b_2 = 1,\ C_0 = 1,\ b_3 = -2,\ \omega \approx -2.15$

(f)

$b_1 = 1,\ a = 1,\ b_2 = 1,\ C_0 = 1,\ b_3 = 1,\ \omega \approx -2.86$

Fig. 2: Phase portraits for the first case

5. $D_1 = 0,\ \ -b_1/a > 0$. Eq. (25) has two equilibria y_{1s} and y_{2s} out of which one is a saddle and one is degenerate. On the curve $D_1 = 0$ in the parameter space a pitchfork bifurcation occurs, where two equilibria of Eq. (25) either appear or vanish.
6. $D_1 = 0,\ \ -b_1/a < 0$. Eq. (25) has two equilibria y_{1s} and y_{2s} out of which one is a center and one is degenerate. On the curve $D_1 = 0$ in the parameter space a pitchfork bifurcation occurs, where two equilibria of Eq. (25) either appear or vanish.

The phase portraits for the above cases are shown in Figs. 1-2. The pitchfork bifurcationw occur in the last two instances of the phase portraits.

Next, we write the equation studied for $n = 1$, $C_1 = 0$, $b_2 = 0$ and $b_3 = 0$

$$y_z = v, \ \ v_z = -\frac{b_1}{a} y^3 - \left(\frac{\omega}{a} + \frac{C_0^2}{4a}\right) y - \frac{b_4}{ay}. \tag{30}$$

The right hand side of the system of equations (30) is not continuous at $y = 0$. To get the regular system associated to (30), we use the following variable transformation

$$dz = ayd\xi. \tag{31}$$

Therefore, the regular system assosiated with (30) is written as follows

$$y_\xi = ayv, \ \ v_\xi = -b_1 y^4 - \left(\omega + \frac{C_0^2}{4}\right) y^2 - b_4 \equiv f_2(y) \tag{32}$$

Systems of equations (30) and (32) have the same first integral

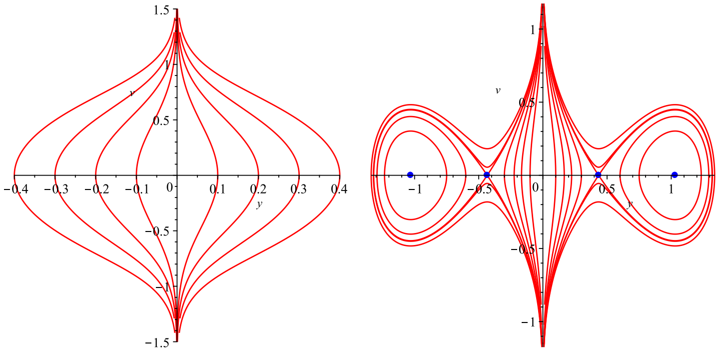
$$H_2(y, v) = \frac{v^2}{2} + \frac{b_1}{a}\frac{y^4}{4} + \left(\frac{\omega}{a} + \frac{C_0^2}{4a}\right)\frac{y^2}{2} + \frac{b_4}{a}\ln y, \tag{33}$$

therefore their orbits are topologically the same, with the exception of the line $y = 0$. Thus, we can investigate the stability of equilibria of (32), which will match the stability of equilibria of (30). Every equilibrium of (32) is located on the v axis with the coordinate determined by the following equation

$$b_1 y^4 + \left(\omega + \frac{C_0^2}{4}\right) y^2 + b_4 = -f_2(y) = 0. \tag{34}$$

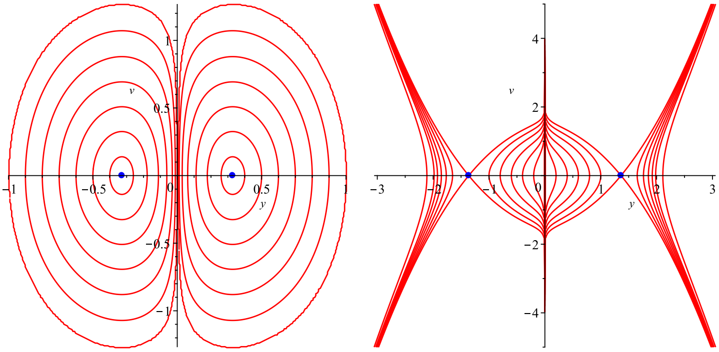
This equation can have at most four roots. Let us denote them as $y_{1s},\ y_{2s},\ y_{3s},\ y_{4s}$. Provided that the root is real, the stability of an equilibrium point $(y_{is}, 0)$ is determined by the eigenvalues of the matrix

$$\begin{pmatrix} 0 & ay_{is} \\ -\frac{df_2}{dy}\big|_{y_{is}} & 0 \end{pmatrix}, \ \ i = 1..3. \tag{35}$$



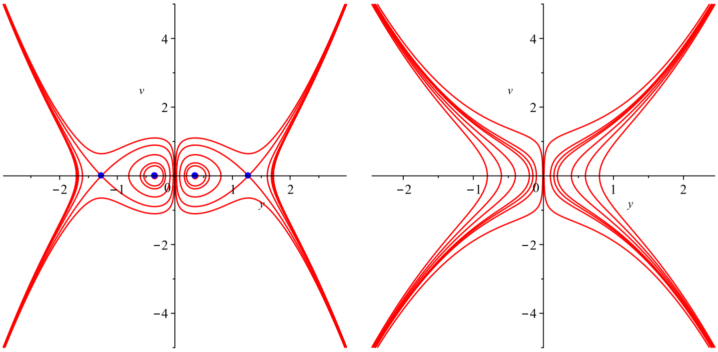
(a) $b_1 = 1,\ a = 1, b_4 = 0.2,\ C_0 = 1,\ \omega = 3/2$

(b) $b_1 = 1,\ a = 1, b_4 = 0.2,\ C_0 = 1,\ \omega = -3/2$



(c) $b_1 = 1,\ a = 1, b_4 = -0.2,\ C_0 = 1,\ \omega = -3/2$

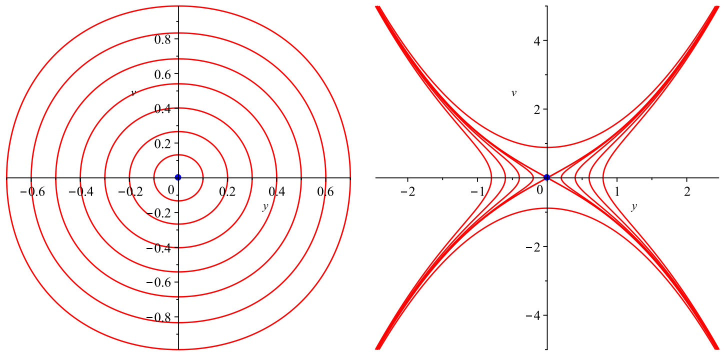
(d) $b_1 = -1,\ a = 1, b_4 = 0.2,\ C_0 = 1,\ \omega = 3/2$



(e) $b_1 = -1,\ a = 1,\ b_4 = -0.2,\ C_0 = 1,\ \omega = 3/2$

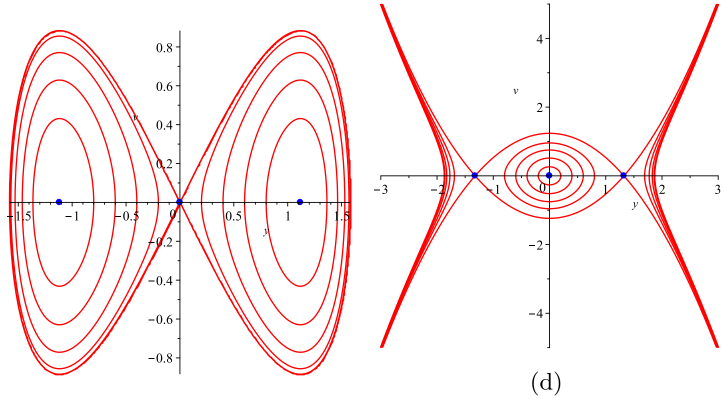
(f) $b_1 = -1,\ a = 1,\ b_4 = -0.2,\ C_0 = 1,\ \omega = -3/2$

Fig. 3: Phase portraits for the second case



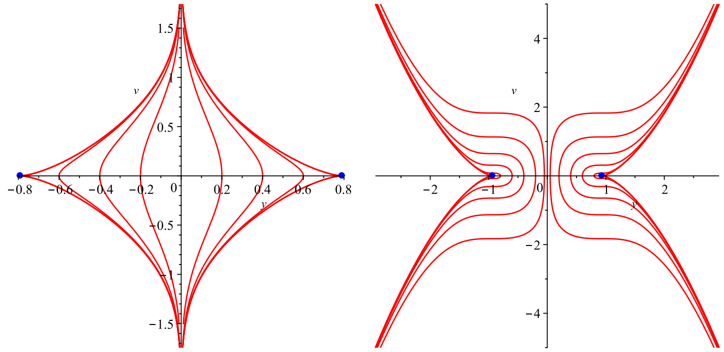
(a) (b)

$b_1 = 1,\ a = 1, b_4 = 0,\ C_0 = 1,$ $b_1 = -1,\ a = 1, b_4 = 0,\ C_0 = 1,$
$\omega = 3/2$ $\omega = -3/2$



(d)

(c) $b_1 = 1,\ a = 1, b_4 = 0,\ C_0 = 1,$ $b_1 = -1,\ a = 1, b_4 = 0,\ C_0 = 1,$
$\omega = -3/2$ $\omega = 3/2$



(e) $b_1 = 1,\ a = 1,$ (f) $b_1 = -1,\ a = 1,$
$b_4 = 25/64,\ C_0 = 1,\ \omega = -3/2$ $b_4 = -49/64,\ C_0 = 1,\ \omega = 3/2$

Fig. 4: Phase portraits for the second case

Accordingly, if $f_2(y)$ increases at the point y_s and $y_s > 0$, or decreases at the point y_s and $y_s < 0$, then an equilibrium is a saddle; if $f_2(y)$ decreases at y_s and $y_s > 0$, or $f_2(y)$ increases at y_s and $y_s < 0$, then it is a center, otherwise it is a degenerate equilibrium (this works for $a > 0$, in the case of $a < 0$ the situation is reversed). Thus, we present the following classification for equilibrium points in this case (in all the cases we assume that the discriminant of Eq. (34) is nonnegative $D_2 \geq 0$, except for the last one)

1. $b_1 > 0,\ \ b_4 > 0,\ \ \omega + C_0^2/4 \geq 0$. Eq. (32) has no equilibria (Fig. 3a).
2. $b_1 > 0,\ \ b_4 > 0,\ \ \omega + C_0^2/4 < 0$. Eq. (32) has four equilibria in a sequence: (center, saddle, saddle, center) (Fig. 3b).
3. $b_1 > 0,\ \ b_4 < 0$. Eq. (32) has two center equilibria (Fig. 3c).
4. $b_1 < 0,\ \ b_4 > 0$. Eq. (32) has two saddle equilibria (Fig. 3d).
5. $b_1 < 0,\ \ b_4 < 0,\ \ \omega + C_0^2/4 > 0$. Eq. (32) has four equilibria in a sequence: (saddle, center, center, saddle) (Fig. 3e).
6. $b_1 < 0,\ \ b_4 < 0,\ \ \omega + C_0^2/4 < 0$. Equation (32) has no equilibria (Fig. 3f).
7. $b_1 \cdot (\omega + C_0^2/4) > 0,\ \ b_4 = 0$. Eq. (32) has a degenerate zero equilibrium (Fig. 4a-4b).
8. $b_1 > 0,\ \ \omega + C_0^2/4 < 0,\ \ b_4 = 0$. Eq. (32) has three equilibria in a sequence: (center, degenerate, center) (Fig. 4c).
9. $b_1 < 0,\ \ \omega + C_0^2/4 > 0,\ \ b_4 = 0$. Eq. (32) has three equilibria in a sequnce: (saddle, degenerate, saddle) (Fig. 4d).
10. $b_1 \cdot (\omega + C_0^2/4) < 0,\ \ D_2 = 0$. Eq. (32) has two degenerate equilibria (Fig. 4e).

4 The first conservation law corresponding to Eq. (1)

Let us write Eq. (1) as the system of equations in the form

$$i\, q_t + a\, q_{xx} + \left(b_1\, |q|^{2n} + b_2\, |q|^{n} + b_3\, |q|^{-n} + b_4\, |q|^{-2n}\right)\, q = 0, \tag{36}$$

and

$$-i\, q_t^* + a\, q_{xx}^* + \left(b_1\, |q|^{2n} + b_2\, |q|^{n} + b_3\, |q|^{-n} + b_4\, |q|^{-2n}\right)\, q^* = 0. \tag{37}$$

Multiplying Eq. (36) by q^* and Eq. (37) by $-q$ and adding the resulting expressions, we obtain the equation

$$i\,(q^*\, q_t + q\, q_t^*) + a\,(q^* q_{xx} - q\, q_{xx}^*) = 0. \tag{38}$$

Eq. (38) can be presented in the form

$$\frac{\partial T_1}{\partial t} + \frac{\partial X_1}{\partial x} = 0. \tag{39}$$

where T_1 and X_1 take the form

$$T_1 = i\,|q|^2, \qquad X_1 = a\,(q^* q_x - q\,q_x^*). \tag{40}$$

From Eq. (39) follows the conservative quantity

$$P = \int_{-\infty}^{\infty} |q|^2\,dx = Const. \tag{41}$$

The conservative quantity (41) correspons to the impulse power.

5 The second conservation law corresponding to Eq. (1)

The second conservation law can be found by multiplying Eq. (36) by q_x^* and Eq. (37) by q_x and adding the resulting equations. We get

$$i\,(q_x^*\,q_t - q_x\,q_t^*) + a\,(q_x^*\,q_{xx} + q_x\,q_{xx}^*) + b_1\,|q|^{2n}\,(q_x^*\,q + q_x\,q^*) +$$

$$b_2\,|q|^n\,(q_x^*\,q + q_x\,q^*) + b_3\,|q|^{-n}\,(q_x^*\,q + q_x\,q^*) + b_4\,|q|^{-2n}\,(q_x^*\,q + q_x\,q^*) = 0. \tag{42}$$

Taking into account the following formulas

$$q_x^*\,q_t - q_x\,q_t^* = \frac{1}{2}\frac{\partial}{\partial t}\,(q_x^* q - q^* q_x) - \frac{1}{2}\frac{\partial}{\partial x}\,(q_t^* q - q^* q_t), \tag{43}$$

$$q_x^* q + q^* q_x = \frac{\partial}{\partial x}\,(|q|^2), \tag{44}$$

$$q_x^* q_{xx} + q_{xx}^* q_x = \frac{\partial}{\partial x}\,(|q_x|^2), \tag{45}$$

We can write Eq. (42) in the form

$$\frac{i}{2}\frac{\partial}{\partial t}\,(q_x^* q - q^* q_x) - \frac{i}{2}\frac{\partial}{\partial x}\,(q_t^* q - q^* q_t) + a\,\frac{\partial}{\partial x}\,(|q_x|^2) +$$

$$\frac{2b_1}{2n+2}\frac{\partial}{\partial x}|q|^{2n+2} + \frac{2b_2}{n+2}\frac{\partial}{\partial x}|q|^{n+2} + \tag{46}$$

$$\frac{2b_3}{2-n}\frac{\partial}{\partial x}|q|^{2-n} + \frac{2b_4}{2-2n}\frac{\partial}{\partial x}|q|^{2-2n}.$$

The last equation can be presented as the conservation law

$$\frac{\partial T_2}{\partial t} + \frac{\partial X_2}{\partial x} = 0, \tag{47}$$

where T_2 and X_2 are determined by formulas

$$T_2 = \frac{i}{2}\,(q_x^* q - q^* q_x), \tag{48}$$

$$\begin{gathered} X_2 = \frac{i}{2}\,(q^* q_t - q_t^* q) + a\,|q_x|^2 + \frac{2b_1}{2n+2}\,|q|^{2n+2} + \frac{2b_2}{n+2}\,|q|^{n+2} + \\ \frac{2b_3}{2-n}\,|q|^{2-n} + \frac{2b_4}{2-2n}\,|q|^{2-2n}. \end{gathered} \tag{49}$$

From Eq. (47) we obtain the conservative quantity in the form

$$M = \frac{i}{2}\int_{-\infty}^{\infty}(q_x^* q - q^* q_x) = Const. \tag{50}$$

Conservative quantity (50) corresponds to the conservation of the momentum of the solution $q(x,t)$.

6 The third conservation law corresponding to Eq. (1)

At the first step we multiply Eq. (36) by $|q|^{2n}\,q^*$ and Eq. (37) by $-|q|^{2n}\,q$. After that we add the equations obtained. As a result we have the following equation

$$\frac{2\,i}{2n+2}\,\frac{\partial\,|q|^{2n+2}}{\partial t} + a\,|q|^{2n}\,(q^*\,q_{xx} - q\,q_{xx}^*) = 0. \tag{51}$$

We also have the following equation after multiplying Eq. (36) by $|q|^n\,q^*$ and Eq. (37) by $-|q|^n\,q$ and then adding them. We get

$$\frac{2\,i}{n+2}\,\frac{\partial\,|q|^{n+2}}{\partial t} + a\,|q|^{n}\,(q^*\,q_{xx} - q\,q_{xx}^*) = 0. \tag{52}$$

The two following equations can be obtained by multiplying Eq. (36) by $|q|^{-n}\,q^*$ and by $|q|^{-2n}\,q^*$, consequently, and Eq. (37) by $-|q|^{-n}\,q$, and bt $-|q|^{-2n}\,q$. Adding these expressions yields two following equations

$$\frac{2\,i}{2-n}\,\frac{\partial\,|q|^{2-n}}{\partial t} + a\,|q|^{-n}\,(q^*\,q_{xx} - q\,q_{xx}^*) = 0 \tag{53}$$

and

$$\frac{2\,i}{2-2n}\,\frac{\partial\,|q|^{2-2n}}{\partial t} + a\,|q|^{-2n}\,(q^*\,q_{xx} - q\,q_{xx}^*) = 0. \tag{54}$$

From Eqs. (51) - (54) one can see that we need other equations to find the third conservation law of Eq. (1). At the second step, first of all, we multiply

Eq. (36) by q^*_{xx} and Eq. (37) by $-q_{xx}$. Adding the expressions obtained, we have the equation

$$i\,\frac{\partial}{\partial x}\left(q_x^*\,q_t+q_x\,q_t^*\right)-i\,\frac{\partial\,|q_x|^2}{\partial t}+\left(q\,q_{xx}^*-q_{xx}\,q^*\right)\left(b_1\,|q|^{2n}+\right.$$
$$\left.b_2\,|q|^{n}+b_3\,|q|^{-n}+b_4\,|q|^{-2n}\right)=0. \tag{55}$$

At the third step, we, first of all, multiply Eq. (51), Eq. (52), Eq. (53) and Eq. (54) by b_1, b_2, b_3 and b_4, respectively. Then, Eq. (55) is multiplied by a. Adding five equations obtained yields the equation in the form

$$\frac{2\,i\,b_1}{2n+2}\,\frac{\partial\,|q|^{2n+2}}{\partial t}+\frac{2\,i\,b_2}{n+2}\,\frac{\partial\,|q|^{n+2}}{\partial t}+\frac{2\,i\,b_3}{2-n}\,\frac{\partial\,|q|^{2-n}}{\partial t}+$$
$$\frac{2\,i\,b_4}{2-2n}\,\frac{\partial\,|q|^{2-2n}}{\partial t}-i\,a\frac{\partial\,|q_x|^2}{\partial t}+i\,a\,\frac{\partial}{\partial x}\left(q_x^*\,q_t+q_x\,q_t^*\right). \tag{56}$$

The last equation can be written as the conservation law

$$\frac{\partial\,T_3}{\partial t}+\frac{\partial\,X_3}{\partial x}=0, \tag{57}$$

where T_3 and X_3 take the form

$$T_3=\frac{2\,b_1\,|q|^{2n+2}}{2n+2}+\frac{2\,b_2\,|q|^{n+2}}{n+2}+\frac{2\,b_3\,|q|^{2-n}}{2-n}+\frac{2\,b_4\,|q|^{2-2n}}{2-2n}-a\,|q_x|^2 \tag{58}$$

and

$$X_3=a\,\left(q_x^*\,q_t+q_x\,q_t^*\right). \tag{59}$$

From (57) we obtain the conservative quantity in the form

$$H=\int_{-\infty}^{\infty}\left(\frac{2\,b_1}{2\,n+2}\,|q|^{2n+2}+\frac{2\,b_2}{n+2}\,|q|^{n+2}+\frac{2\,b_3}{2-n}\,|q|^{2-n}+\right.$$
$$\left.\frac{2\,b_4}{2-2n}\,|q|^{2-2n}-a\,|q_x|^2\right)\,dx=Const. \tag{60}$$

Expression (60) corresponds to the conservation of energy for the optical soliton of Eq. (1).

7 Conservative quantities corresponding to the soliton (23) of Eq. (1)

Using the conservation laws we can calculate conservative quantities of solutions of Eq. (1). Without loss of generality, let us calculate the conservative quantities of optical soliton (23) corresponding to Eq. (1).

Let us note that to calculate the conservative quantity we use the following integral (Hammer 1953)

$$\Omega(\rho, m, k) = \int_0^\infty \frac{x^{2k-1}}{\left(1 + 2\,\rho\, x + x^2\right)^{2m}}\, dx = \tag{61}$$
$$(\rho - \sqrt{\rho^2 - 1})^{2\,k}\, B(4m - 2k, 2k)\, F(2\,k, 2\,m, 4\,m, \frac{2\,\sqrt{\rho^2 - 1})}{\rho + \sqrt{\rho^2 - 1})}),$$

where $B(x, y)$ is the beta function and $F(a, b, c, z)$ is the Gaussian hypergeometric function, and $2m > k$.

Substituting (23) into (41), we obtain the power of optical soliton (23) in the form

$$P = \int_{-\infty}^{\infty} \left[\frac{4\,\mu}{2\,\beta + (\beta^2 - 4\,\mu\,\nu)\, \mathrm{e}^{-\sqrt{\mu} z} + \mathrm{e}^{\sqrt{\mu} z}}\right]^{\frac{2}{n}} dz. \tag{62}$$

Using the new variable $\xi = \frac{1}{\sqrt{\mu}} \ln(z)$, the integral (41) is reduced to the following

$$P = \frac{(4\,\mu)^{\frac{2}{n}}}{\sqrt{\mu}} \int_0^\infty \frac{\xi^{\frac{2}{n}-1}}{\left(\beta^2 - 4\,\mu\,\nu + 2\,\beta\,\xi + \xi^2\right)^{\frac{2}{n}}}\, d\xi = \frac{(4\,\mu)^{\frac{2}{n}}}{\sqrt{\mu}\,(\beta^2 - 4\mu\nu)^{\frac{1}{n}}} \tag{63}$$
$$\Omega\left(\frac{\beta}{\sqrt{\beta^2 - 4\,\mu\,\nu}}, \frac{1}{n}, \frac{1}{n}\right).$$

The conservative quantity corresponding to the momentum is found by substituting solution (23) into expression (50). As a result we have

$$M = \frac{C_0\,(4\,\mu)^{\frac{2}{n}}}{2\,a\,\sqrt{\mu}\,(\beta^2 - 4\,\mu\,\nu)^{\frac{1}{n}}}\, \Omega\left(\frac{\beta}{\sqrt{\beta^2 - 4\,\mu\,\nu}}, \frac{1}{n}, \frac{1}{n}\right). \tag{64}$$

Conservative quantity of solution (23) corresponding to Eq. (1) can be calculated by substituting solution (23) into (60) and taking into account integral

(61) at $b_3 = 0$ and $b_4 = 0$. This yields the conservative quantity in the form

$$
\begin{gathered}
H = \frac{2\, b_1\, (4\,\mu)^{\frac{2n+2}{n}}}{(2n+2)\sqrt{\mu}\,(\beta^2 - 4\mu\nu)^{\frac{n+1}{n}}}\;\Omega\left(\frac{\beta}{\sqrt{\beta^2 - 4\,\mu\,\nu}},\, 1+\frac{1}{n},\, 1+\frac{1}{n}\right)+ \\
\frac{2\, b_2\, (4\,\mu)^{\frac{n+2}{n}}}{(n+2)\sqrt{\mu}\,(\beta^2 - 4\mu\,\nu)^{\frac{n+2}{2n}}}\;\Omega\left(\frac{\beta}{\sqrt{\beta^2 - 4\,\mu\,\nu}},\, \frac{1}{2}+\frac{1}{n},\, \frac{1}{2}+\frac{1}{n}\right)- \\
\frac{C_0^2}{4\,a}\frac{(4\,\mu)^{\frac{2}{n}}}{\sqrt{\mu}\,(\beta^2 - 4\mu\nu)^{\frac{1}{n}}}\;\Omega\left(\frac{\beta}{\sqrt{\beta^2 - 4\,\mu\,\nu}},\, \frac{1}{n},\, \frac{1}{n}\right)- \\
\frac{4^{\frac{2}{n}}\,\mu^{\frac{2}{n}+\frac{1}{2}}\,a}{n^2\,(\beta^2 - 4\mu\nu)^{\frac{1}{n}}}\,(\Omega(\frac{\beta}{\sqrt{\beta^2 - 4\,\mu\,\nu}},\, 1+\frac{1}{n},\, 2+\frac{1}{n})- \\
2\,\Omega(\frac{\beta}{\sqrt{\beta^2 - 4\,\mu\,\nu}},\, 1+\frac{1}{n},\, 1+\frac{1}{n})+ \\
\Omega(\frac{\beta}{\sqrt{\beta^2 - 4\,\mu\,\nu}},\, 1+\frac{1}{n},\, \frac{1}{n})).
\end{gathered}
\tag{65}
$$

This conservative quantity (65) corresponds to the energy of the optical soliton (23).

Conclusion

In this paper we have studied the mathematical model for propagation pulses with four powers of nonlinearity. The Cauchy problem for this equation cannot be solved by the inverse scattering transform. However, this nonlinear partial differential equation is the generalization of the nonlinear Schrödiner equation and some other the well-known mathematical models for description of propagation pulses in optical medium, therefore it allows us to evaluate the influence of various processes on pulse propagation. The main problem of this paper was the construction of conservation laws of Eq. (1). It has been shown that there are three conservation laws corresponding to Eq. (1). Analytical solutions of the generalized nonlinear Schrödinger equation (1) are presented. Optical solitons corresponding to the mathematical model have been given. Conservative quantities for the bright optical soliton have been calculated.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that credit have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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