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Incoherent X-ray diffraction radiation for noninvasive diagnostics of transverse size of ultrarelativistic beams

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Abstract. Polarization radiation, which includes diffraction radiation, transition radiation, Smith-Purcell radiation, and others, can be a good instrument for beam diagnostics. All information about the beam size is contained in the so-called form-factor of the beam. The form-factor represents the sum of two parts corresponding to the coherent and incoherent radiation. Contrary to the general opinion the incoherent part does not always equal unity. In this report we give theoretical description of the incoherent part of the form-factor both for Gaussian and uniform distribution of the ultrarelativistic particles in the bunch. We show that the incoherent form-factor depends on the transverse size of the beam, and discuss the new opportunities for non-invasive diagnostics of ultrarelativistic electron beams.

1. Introduction

Diagnostics of ultrarelativistic electron beams at modern accelerators, such as colliders, synchrotrons, free electron lasers imposes increased requirements on the emittance. It means that the diagnostics of ultrarelativistic beams should be noninvasive, in order to keep the state of the beam in the process of diagnostics. From that point of view, popular today diagnostics schemes based on wire-scanning, scintillator or transition radiation screens, are less preferable than the schemes providing diagnostics without direct scattering of bunches on the target material, such as diffraction radiation (DR) or Smith-Purcell radiation (SPR).

On the other hand, all traditional diagnostics schemes face with the problem of coherence due to very short lengths of electron bunches used in modern accelerators. This problem of undesirable coherence can be solved by the shifting the wavelengths of radiation to EUV and X-ray part of spectrum [1]. Also, such scheme has the advantage that the less wavelength of radiation, the more accuracy one can reach, according to the Rayleigh limit.

In this research we suggest new scheme based on the theory of EUV and X-ray DR [2], which provides noninvasive diagnostics, and, owing to the short wavelengths, submicron accuracy.

2. Spectral-angular distribution of incoherent radiation

In general case of DR or SPR the spectral-angular distribution of radiation has the form [3, 4]:

$$\frac{dW(\mathbf{n}, \omega)}{d\omega d\Omega} = \frac{dW_0(\mathbf{n}, \omega)}{d\omega d\Omega} \frac{\sin^2(Nd\varphi/2)}{\sin^2(d\varphi/2)} F, \quad (1)$$



where $dW_0/d\omega d\Omega$ is the spectral-angular distribution of radiation from a single particle and a single plate, d is the period of the grating, N is the number of the plates, F is the form-factor [4-8]:

$$F = N_e F_{inc}(\mathbf{n}, \omega) + N_e(N_e - 1) F_{tr}(\mathbf{n}, \omega) F_l(\omega), \quad (2)$$

N_e is the number of particles, $F_{inc}(\mathbf{n}, \omega)$ is incoherent form-factor, $F_{tr}(\mathbf{n}, \omega)$ and $F_l(\omega)$ are the transverse and longitudinal form-factors, which depend only on transverse or longitudinal sizes and shape of the beam, $\mathbf{n} = (n_x, n_y, n_z)$ is the unit wave-vector, ω is the frequency, Ω is the solid angle, c is the speed of light, $\beta = \mathbf{v}/c$, $\mathbf{v}$ is the particle's velocity, γ is the Lorentz-factor,

$$\varphi = \beta_x^{-1} (1 - n_x \beta_x - n_y \beta_y) \omega / c. \quad (3)$$

For formulae given above the coordinate system is chosen so that the particle moves along the x -axis, the normal to the surface of the target coincides with the z -axis, see figure 1.

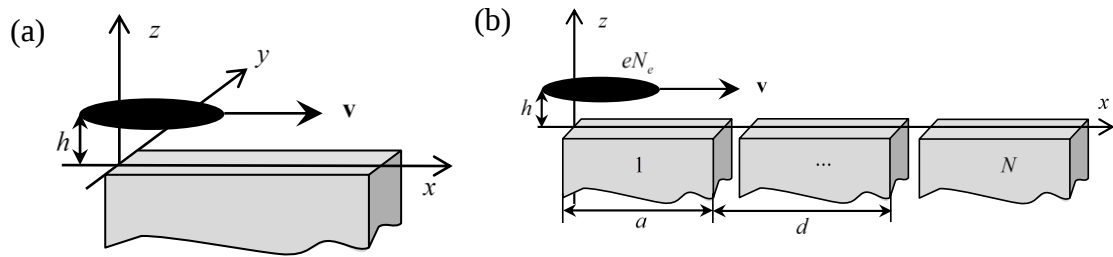


Figure 1. Generation of (a) DR and (b) SPR by the moving bunch near the target.

While the analytical form of $dW_0/d\omega d\Omega$ for optical DR from conducting targets is well-known: there are developed theories, see for example [2, 4, 9 – 11] some of which are also experimentally proved [3], there are no many theories valid for X-ray spectral region.

In X-ray region, i.e. at frequencies $\omega \gg \omega_p$, where ω_p is the plasma frequency, the function of dielectric permittivity has a well-known universal form $\varepsilon(\omega) = 1 - \omega_p^2/\omega^2$.

DR in the high-frequency limit was investigated in the papers [12] for non-relativistic particles, and in [13] for gratings. Non-relativistic particles, however, emits DR only for impact-parameters of the order of the wavelength, which is not the case for X-ray domain. The calculations in the paper of M.J. Moran [13], on the other hand, was based on the theoretical description given in [14] by M.L. Ter-Mikhaelyan, who developed the theory for an infinitely thin perfectly conducting target. This makes the results inapplicable for frequencies larger than optical ones. After that, the basis of the X-ray DR theory was given in [15] and for SPR in [2] for single-particle radiation.

One of the main features of the spectral-angular distribution of the polarization radiation is its dependence on the impact-parameter, i.e. the shortest distance between the moving charge and the target surface; see the parameter h in figure 1. This dependence is

$$\frac{dW(\mathbf{n}, \omega)}{d\Omega d\omega} \propto \exp(-2\rho h), \quad (4)$$

where $\rho = \frac{\omega}{c\beta\gamma} \sqrt{1 + \gamma^2 \beta_x^{-2} (n_y \beta^2 - \beta_y)^2}$. From equation (4) it is clear that the farther the particle from the target surface is, the less intensive the radiation is. The radiation is maximal when the argument of the exponent in equation (4) is minimal, i.e. when ρ is minimal:

$$n_y = \beta_y / \beta^2. \quad (5)$$

Condition (5) defines the angles at which it is expedient to investigate the radiation properties. Testing the ratio of sines in equation (1) for maxima it is easy to obtain the dispersion relation:

$$d(1 - n_x \beta_x - n_y \beta_y) = \lambda \beta_x m, \quad m = 1, 2, \dots, \quad (6)$$

which coincides with the previous results [16, 17].

Usually the incoherent form-factor is supposed to be equal to unity, like it occurs for synchrotron radiation or transition radiation from an infinite media of an infinite slab. For polarization radiation from the target edge (DR, SPR) the incoherent form-factor does not equal unity $F_{inc} \neq 1$.

This fact was explained in detail in the paper [4]. The way to obtain the formula for the coherent and incoherent form-factor was described in [18] for DR and SPR.

If the distribution in the bunch does not depend on the relative position of the particles, then the transverse and longitudinal parts of the form-factor can be written as the separate terms:

$$F_{inc} = F_{inc}(r_0), \quad F_{coh} = F_{tr}(r_0) F_l(l), \quad (7)$$

where l and r_0 are the length and the transverse bunch size. For example, for the cylindrical bunch of the length l and the radius r_0 with the uniform distribution of the particles it is easy to find [4]:

$$F_{inc} = 2 \frac{I_1(2\rho r_0)}{2\rho r_0}, \quad (8)$$

where $I_1(x)$ is the modified Bessel function of the first order. For the Gaussian distribution:

$$f(\mathbf{r}) = \frac{1}{\pi \sqrt{\pi} \sigma_x \sigma_y \sigma_z} \exp \left[-\frac{x^2}{\sigma_x^2} - \frac{y^2}{\sigma_y^2} - \frac{z^2}{\sigma_z^2} \right] \quad (9)$$

the calculations are more difficult and the result can be seen in the paper [19]. In a short form these formulae can be written as:

$$F_{inc} = \frac{1}{2} \exp[\rho^2 \sigma_z^2] (1 - \Phi[\rho \sigma_z - h/\sigma_z]), \quad (10)$$

where $\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ is the Laplace function, $\mathbf{k} = \mathbf{n}\omega/c$ is the wave-vector. It may seem that these exponents increase indefinitely with $\omega \rightarrow \infty$, which is contained in ρ . However, using well-known asymptotic form of Laplace function for $x \rightarrow \infty$ one can show that F_{coh} and F_{inc} decrease with growing of frequency.

From equations (8) and (10) it is seen that the transverse size of the beam can be detected from incoherent radiation. The procedure is the same as for longitudinal one. For the analysis let us define the polar and azimuthal angles θ and ϕ as $\mathbf{n} = (\sin \theta \cos \phi, \cos \theta, \sin \theta \sin \phi)$.

Below we will consider the incoherent form-factor taking into account the exponent in equation (4) because it is the term which strongly influence the radiation intensity and properties, while all other terms, contained in the distribution of energy of radiation from a single particle $dW_0/d\omega d\Omega$, do not.

In figure 2 the form-factors multiplied by the exponent from in equation (4) both for the uniform distribution (black curves) and for the Gaussian distribution (red dashed curves) are shown in dependence on the wavelength of radiation. It is seen from figure 2 that the difference in the shape of the bunch for incoherent radiation influences the distribution significantly.

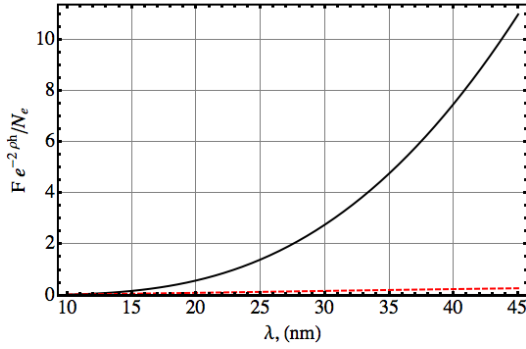


Figure 2. Form-factors normalized to N for incoherent radiation ($\sigma_x \gg \lambda$) for the uniform distribution (black curves) and for the Gaussian distribution (red dashed curves) of the particles in the bunch. Here $\gamma = 10^4$, $h = 50 \mu\text{m}$, $\phi = \gamma^{-1}$, $\theta = 0$, $\sigma_y = \sigma_z = 40 \mu\text{m}$, $r_0 = \sigma_z$.

The dependences of the form-factors on the transverse size of the bunch $r_0 = \sigma_z = \sigma_y$ is shown in figure 3. Here the black curve is for the uniform distribution, and the red dashed curve is for the Gaussian one. The possibility to define the characteristic beam sizes is seen from the figure 3.

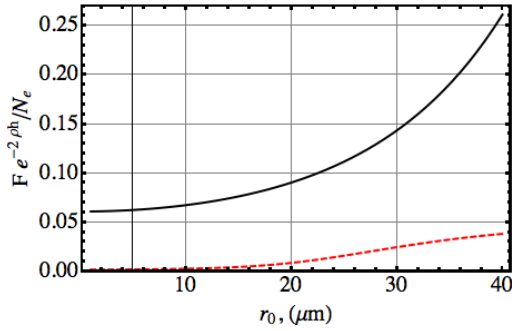


Figure 3. Form-factors for the uniform distribution (black curves) and for the Gaussian distribution (red dashed curves) of the particles in the bunch normalized to N_e depending on the beam radius r_0 , $\lambda = 10 \text{ nm}$, $l = 40 \mu\text{m}$. All other parameters are the same as in figure 2.

The distribution of the SPR generated by a cylindrical and Gaussian bunch is shown in figures 4a and 4b. All black solid curves are plotted for $\alpha = 3^\circ$ and $r_0 = 20 \mu\text{m}$; all blue dashed curves are for $\alpha = 23^\circ$ and $r_0 = 20 \mu\text{m}$; all red dotted curves are for $\alpha = 3^\circ$ and $r_0 = 55 \mu\text{m}$. The graphs were plotted taking into account equation (5). It is seen that in principle the oblique passage leads to the effects, which can be distinguished from the effect of beam transverse size during the experiment.

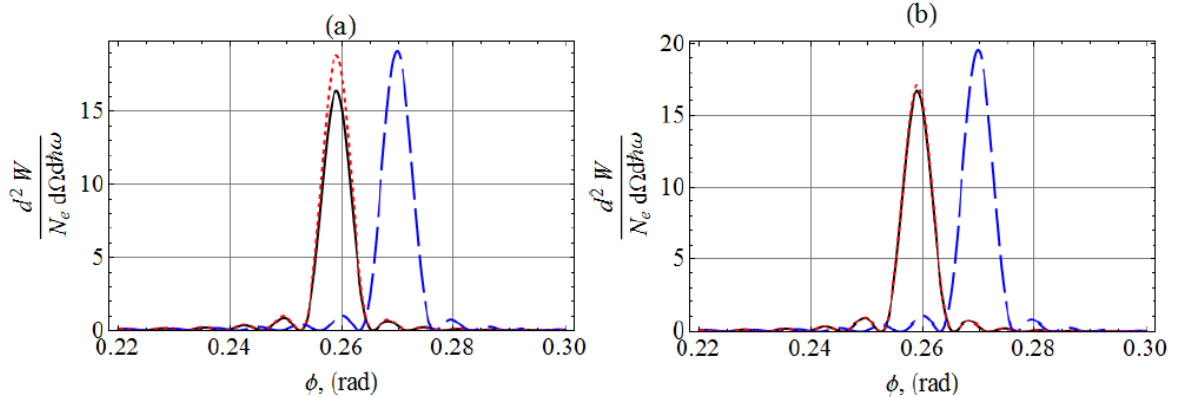


Figure 4. The distribution of X-Ray SPR over the conical surface at $\theta = \arccos(\beta^{-1} \sin \alpha)$. Here $\gamma = 2 \cdot 10^4$, $\hbar \omega_p = 26.1 \text{ eV}$ (beryllium), $d = 0.9 \mu\text{m}$, $a = 0.45 \mu\text{m}$, $\lambda = 30 \text{ nm}$, $h = 60 \mu\text{m}$, $N = 20$, all black solid curves are plotted for $\alpha = 3^\circ$ and $r_0 = 20 \mu\text{m}$; all blue dashed curves are plotted for $\alpha = 23^\circ$ and $r_0 = 20 \mu\text{m}$; all red dotted curves are plotted for $\alpha = 3^\circ$ and $r_0 = 55 \mu\text{m}$.

If we suppose that the particles in the bunch moves at different angles to the general direction of the bunch, then equation (8) will contain the factor describing the velocity distribution, or even information about the divergence of the beam [20].

3. Conclusion

Thus, the theory developed describes the diffraction radiation of electron ultra-relativistic beams in X-ray frequency domain. The obtained expressions describe the intensity of radiation as a function of the beam size, i.e., measuring the intensity one can retrieve the information about the bunch.

Note that X-ray SPR is emitted at the frequencies up to $\omega = \gamma c/2h$, which for the electron energies of the order of 1 TeV can reach the photon energies up to some 30-40 keV. Manufacturing the gratings with a period from some hundreds to tens of nanometres, we can obtain the SPR peaks under the angles from 3-4 to some tens of degrees, which is suitable for the experimental diagnostics schemes. The expression for spectral-angular distribution, generalizing the one obtained in [15], contains the information about both the target and beam parameters, including the length of the beam, which opens new possibility of submicron non-invasive diagnostics for the future electron accelerators and colliders. In contrast with the optical range based diagnostics schemes, X-ray range makes it possible the non-invasive diagnostics with submicron and even nanometres accuracy.

Acknowledgments

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