Results in Physics 16 (2020) 102913

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

Cubic-quartic optical solitons in birefringent fibers with four forms of nonlinear refractive index by exp-function expansion

Yakup Yıldırım[a], Anjan Biswas[b,c,d,e], Anwar Ja'afar Mohamad Jawad[f], Mehmet Ekici[g], Qin Zhou[h,*], Salam Khan[b], Abdullah Kamis Alzahrani[c], Milivoj R. Belic[i]

[a] Department of Mathematics, Faculty of Arts and Sciences, Near East University, 99138 Nicosia, Cyprus
[b] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762-7500, USA
[c] Department of Mathematics, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[d] Department of Applied Mathematics, National Research Nuclear University, 31 Kashirskoe Shosse, Moscow 115409, Russian Federation
[e] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria0008, South Africa
[f] Al-Rafidain University College, Baghdad 10014, Iraq
[g] Department of Mathematics, Faculty of Science and Arts, Yozgat Bozok University, 66100 Yozgat, Turkey
[h] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China
[i] Science Program, Texas A&M University at Qatar, PO Box 23874, Doha, Qatar

ARTICLE INFO

Keywords:
Cubic-quartic solitons
Integrability
Birefringence

ABSTRACT

This paper applies exp-function expansion algorithm to secure to cubic-quartic dark and singular optical solitons in birefringent fibers that are studied with four forms of nonlinear refractive index. These solitons are listed along with their existence criteria and are also classified.

Introduction

The study of solitons has made a lasting impression in the field of optoelectronics. There are several mathematical methodologies that has been successfully applied, that yielded prolific results, to study the dynamics of optical soliton propagation [1–23] as well as in the field of solitary wave dynamics in fluid dynamics [24,25]. In the field of nonlinear fiber optics, the concept of cubic-quartic (CQ) optical solitons first appeared during 2017 and subsequently an avalanche of results ensued [1–4,7–9]. Prior to this, it is only the concept of pure quartic solitons that existed [5,6]. Such CQ solitons was introduced out of pure necessity.

When chromatic dispersion runs low enough to be discarded, it is third-order dispersion (3OD) and fourth-order dispersion (4OD) effects that collectively compensate for this crisis situation. This enables the delicate balance, between the effects of dispersion and self-phase modulation, to be restored so that the transmission of stable solitons across inter-continental distances is rendered possible. However, the effect of birefringence due to differential group delay is unavoidable. Therefore, it is imperative to address this problem by considering CQ solitons in birefringent fibers. A few preliminary results have been reported during 2020 [19]. This paper is thus a sequel to the previously reported results. The current work implements exponential function expansion to secure CQ dark and singular soliton solutions in birefringent fibers having four forms of nonlinear refractive index. These are Kerr law, parabolic law, quadratic-cubic law and non-local law. The details of the soliton extraction process are enumerated in the rest of the paper after a quick recap of the exp-function expansion algorithm.

Exp-function expansion: a quick overview

A quick overview of the method is enumerated as:
Step-1: A model can be taken as

$$S(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0 \tag{1}$$

which is reduced to

$$G(U, U', U'', U''', \cdots) = 0 \tag{2}$$

by using of

* Corresponding author.
E-mail addresses: qinzhou@whu.edu.cn, qzh@whu.edu.cn (Q. Zhou).

https://doi.org/10.1016/j.rinp.2019.102913
Received 26 November 2019; Received in revised form 26 December 2019; Accepted 26 December 2019
Available online 08 January 2020
2211-3797/ © 2019 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (http://creativecommons.org/licenses/BY/4.0/).

$$u(x,t) = U(\vartheta), \quad \vartheta = x - pt. \tag{3}$$

Step-2: The solution of Eq. (2) is:

$$U(\vartheta) = \sum_{i=0}^{N} A_i \{\exp(-V(\vartheta))\}^i. \tag{4}$$

In consequence of using balancing rule in Eq. (2), the number N is obtained. The function $V(\vartheta)$ holds the following equation

$$V'(\vartheta) = \exp(-V(\vartheta)) + S\exp(V(\vartheta)) + R \tag{5}$$

where S and R are constants. The strategic solutions of Eq. (5) are as

$$V(\vartheta) = \ln\left[-\frac{R}{2S} - \frac{\sqrt{\mu}}{2S}\tanh\left(\frac{\sqrt{\mu}}{2}(\vartheta+\vartheta_0)\right)\right], \quad S \neq 0, \quad \mu > 0,$$

$$V(\vartheta) = \ln\left[-\frac{R}{2S} - \frac{\sqrt{\mu}}{2S}\coth\left(\frac{\sqrt{\mu}}{2}(\vartheta+\vartheta_0)\right)\right], \quad S \neq 0, \quad \mu > 0,$$

$$V(\vartheta) = \ln\left[-\frac{R}{2S} + \frac{\sqrt{-\mu}}{2S}\tan\left(\frac{\sqrt{-\mu}}{2}(\vartheta+\vartheta_0)\right)\right], \quad S \neq 0, \quad \mu < 0,$$

$$V(\vartheta) = \ln\left[-\frac{R}{2S} - \frac{\sqrt{-\mu}}{2S}\cot\left(\frac{\sqrt{-\mu}}{2}(\vartheta+\vartheta_0)\right)\right], \quad S \neq 0, \quad \mu < 0 \tag{6}$$

where $\mu = R^2 - 4S$.

Step-3: As long as Eqs. 4,5 are inserted into Eq. (2), we get the strategic equations. Thus, the important results which are imparted by solving these strategic equations are obtained. Moreover, if these important results with Eq. (6) are substituted into Eq. (4), the optical solitons are recovered.

Kerr law

The CQ nonlinear Schrödinger's equation (NLSE) (CQ-NLSE) equation in polarization-preserving fibers with the Kerr law is:

$$iq_t + iaq_{xxx} + bq_{xxxx} + c\,|q|^2 q = 0. \tag{7}$$

The constant a is the coefficient of the 3OD while the constant b is the coefficient of the 4OD. Also, the coefficient c provides the Kerr law of refractive index. Most optical fibers appear with this law nonlinearity.

The CQ-NLSE in birefringent fibers for Kerr law is:

$$iu_t + ia_1u_{xxx} + b_1u_{xxxx} + (c_1\,|u|^2 + d_1\,|v|^2)u = 0,$$

$$iv_t + ia_2v_{xxx} + b_2v_{xxxx} + (c_2\,|v|^2 + d_2\,|u|^2)v = 0. \tag{8}$$

The constants c_j, and d_j, $(j = 1, 2)$ account for self-phase modulation (SPM) and cross-phase modulation (XPM) respectively. Also, the effect of four-wave mixing is discarded.

Solitons for Eq. (8) are recovered with the exp-expansion function methodology. To this end we decompose the dependent variables into phase-amplitude form as

$$u(x,t) = U_1(\vartheta)e^{i\varphi_1(x,t)},$$

$$v(x,t) = U_2(\vartheta)e^{i\varphi_2(x,t)}. \tag{9}$$

In Eq. (9), the amplitude component is shown as $U_j(\vartheta)$ and the velocity is given by

$$\vartheta = x - pt \tag{10}$$

while the phase component is as

$$\varphi_j(x,t) = -k_jx + w_jt + \zeta_j \tag{11}$$

where the parameters k_j, ζ_j, w_j sequentially correspond to the frequency, phase constant and wave number.

If Eq. (9) is inserted into Eq. (8), we arrive at

$$b_jU_j^{(4)} + (3a_jk_j - 6b_jk_j^2)U_j'' - (w_j + a_jk_j^3 - b_jk_j^4)U_j + c_jU_j^3 + d_jU_jU_{\bar{j}}^2 = 0, \tag{12}$$

$$(a_j - 4b_jk_j)U_j'' - (p + 3a_jk_j^2 - 4b_jk_j^3)U_j = 0 \tag{13}$$

which emerges from real and imaginary parts. These read:

$$b_jU_j^{(4)} + (3a_jk_j - 6b_jk_j^2)U_j'' - (w_j + a_jk_j^3 - b_jk_j^4)U_j + (c_j + d_j)U_j^3 = 0, \tag{14}$$

$$a_j = 4b_jk_j, \tag{15}$$

$$p = -3a_jk_j^2 + 4b_jk_j^3 \tag{16}$$

by using of $U_{\bar{j}} = U_j$.

The balancing rule applied to Eq. (14), Eq. (4) gives

$$U_j = A_0 + A_1\exp(-V(\vartheta)) + A_2\exp(-2V(\vartheta)). \tag{17}$$

After Eqs. (5) and (17) are inserted into Eq. (14), we recover

$$-6RSA_1b_jk_j^2 + 3RSA_1a_jk_j + R^3SA_1b_j + 14R^2S^2A_2b_j + 8RS^2A_1b_j - 12S^2A_2b_jk_j^2 + 6S^2A_2a_jk_j + 16S^3A_2b_j + A_0b_jk_j^4 - A_0a_jk_j^3 - A_0w_j + A_0^3c_j + A_0^3d_j = 0, \tag{18}$$

$$30R^3SA_2b_j + 22R^2SA_1b_j + 120RS^2A_2b_j - 6R^2A_1b_jk_j^2 + 3R^2A_1a_jk_j - 12SA_1b_jk_j^2 + 6SA_1a_jk_j + R^4A_1b_j + 16S^2A_1b_j + A_1b_jk_j^4 - A_1a_jk_j^3 + 3A_0^2A_1c_j + 3A_0^2A_1d_j - A_1w_j - 36RSA_2b_jk_j^2 + 18RSA_2a_jk_j = 0, \tag{19}$$

$$232R^2SA_2b_j + 60RSA_1b_j - 24R^2A_2b_jk_j^2 + 12R^2A_2a_jk_j - 18RA_1b_jk_j^2 - 48SA_2b_jk_j^2 + 9RA_1a_jk_j + 24SA_2a_jk_j + 16R^4A_2b_j + 15R^3A_1b_j + 136S^2A_2b_j + A_2b_jk_j^4 - A_2a_jk_j^3 + 3A_0^2A_2c_j + 3A_0^2A_2d_j + 3A_0A_1^2c_j + 3A_0A_1^2d_j - A_2w_j = 0, \tag{20}$$

$$A_2^3c_j + A_2^3d_j + 120A_2b_j,\ 3A_1A_2^2c_j + 3A_1A_2^2d_j + 336RA_2b_j + 24A_1b_j = 0, \tag{21}$$

$$330R^2A_2b_j + 3A_0A_2^2c_j + 3A_0A_2^2d_j + 3A_1^2A_2c_j + 3A_1^2A_2d_j - 36A_2b_jk_j^2 + 60RA_1b_j + 240SA_2b_j + 18A_2a_jk_j = 0, \tag{22}$$

$$130R^3A_2b_j - 60RA_2b_jk_j^2 + 50R^2A_1b_j + 440RSA_2b_j + 30RA_2a_jk_j + 6A_0A_1A_2c_j + 6A_0A_1A_2d_j + A_1^3c_j + A_1^3d_j - 12A_1b_jk_j^2 + 40SA_1b_j + 6A_1a_jk_j = 0. \tag{23}$$

Thus, the important results are

$$R = \pm\sqrt{\frac{6b_jk_j^2 + 20Sb_j - 3a_jk_j}{5b_j}}, \quad A_0 = SA_2, \tag{24}$$

$$A_1 = \pm A_2\sqrt{\frac{6b_jk_j^2 + 20Sb_j - 3a_jk_j}{5b_j}}, \quad c_j = -\frac{A_2^2d_j + 120b_j}{A_2^2}, \tag{25}$$

$$w_j = -\frac{k_j^2(119b_j^2k_j^2 - 119a_jb_jk_j + 36a_j^2)}{25b_j}, \tag{26}$$

$$\mu = R^2 - 4S = -\frac{3k_j(-2b_jk_j + a_j)}{5b_j} \tag{27}$$

which emerges from Eqs. (18)–(23). Moreover, if Eqs. (24)-(27) with Eq. (6) are substituted into Eq. (17), the singular solitons are

$$u(x,t)=\left\{SA_2-\frac{2A_2S\sqrt{\frac{6b_1k_1^2+20Sb_1-3a_1k_1}{b_1}}}{\sqrt{\frac{6b_1k_1^2+20Sb_1-3a_1k_1}{b_1}}+\sqrt{-\frac{3k_1(-2b_1k_1+a_1)}{b_1}}\tanh\left(\sqrt{-\frac{3k_1(-2b_1k_1+a_1)}{20b_1}}(x-(-3a_1k_1^2+4b_1k_1^3)t)\right)}+\frac{20A_2S^2}{\left(\sqrt{\frac{6b_1k_1^2+20Sb_1-3a_1k_1}{b_1}}+\sqrt{-\frac{3k_1(-2b_1k_1+a_1)}{b_1}}\tanh\left(\sqrt{-\frac{3k_1(-2b_1k_1+a_1)}{20b_1}}(x-(-3a_1k_1^2+4b_1k_1^3)t)\right)\right)^2}\right\}\times e^{i\left(-k_1x-\frac{k_1^2(119b_1^2k_1^2-119a_1b_1k_1+36a_1^2)}{25b_1}t+\zeta_1\right)}, \tag{28}$$

$$v(x,t)=\left\{SA_2-\frac{2A_2S\sqrt{\frac{6b_2k_2^2+20Sb_2-3a_2k_2}{b_2}}}{\sqrt{\frac{6b_2k_2^2+20Sb_2-3a_2k_2}{b_2}}+\sqrt{-\frac{3k_2(-2b_2k_2+a_2)}{b_2}}\tanh\left(\sqrt{-\frac{3k_2(-2b_2k_2+a_2)}{20b_2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)}+\frac{20A_2S^2}{\left(\sqrt{\frac{6b_2k_2^2+20Sb_2-3a_2k_2}{b_2}}+\sqrt{-\frac{3k_2(-2b_2k_2+a_2)}{b_2}}\tanh\left(\sqrt{-\frac{3k_2(-2b_2k_2+a_2)}{20b_2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)\right)^2}\right\}\times e^{i\left(-k_2x-\frac{k_2^2(119b_2^2k_2^2-119a_2b_2k_2+36a_2^2)}{25b_2}t+\zeta_2\right)} \tag{29}$$

with

$b_jk_j(-2b_jk_j+a_j)<0.$

Dark solitons are

$$u(x,t)=\left\{SA_2-\frac{2A_2S\sqrt{\frac{6b_1k_1^2+20Sb_1-3a_1k_1}{b_1}}}{\sqrt{\frac{6b_1k_1^2+20Sb_1-3a_1k_1}{b_1}}+\sqrt{-\frac{3k_1(-2b_1k_1+a_1)}{b_1}}\coth\left(\sqrt{-\frac{3k_1(-2b_1k_1+a_1)}{20b_1}}(x-(-3a_1k_1^2+4b_1k_1^3)t)\right)}+\frac{20A_2S^2}{\left(\sqrt{\frac{6b_1k_1^2+20Sb_1-3a_1k_1}{b_1}}+\sqrt{-\frac{3k_1(-2b_1k_1+a_1)}{b_1}}\coth\left(\sqrt{-\frac{3k_1(-2b_1k_1+a_1)}{20b_1}}(x-(-3a_1k_1^2+4b_1k_1^3)t)\right)\right)^2}\right\}\times e^{i\left(-k_1x-\frac{k_1^2(119b_1^2k_1^2-119a_1b_1k_1+36a_1^2)}{25b_1}t+\zeta_1\right)}, \tag{30}$$

$$v(x,t)=\left\{SA_2-\frac{2A_2S\sqrt{\frac{6b_2k_2^2+20Sb_2-3a_2k_2}{b_2}}}{\sqrt{\frac{6b_2k_2^2+20Sb_2-3a_2k_2}{b_2}}+\sqrt{-\frac{3k_2(-2b_2k_2+a_2)}{b_2}}\coth\left(\sqrt{-\frac{3k_2(-2b_2k_2+a_2)}{20b_2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)}+\frac{20A_2S^2}{\left(\sqrt{\frac{6b_2k_2^2+20Sb_2-3a_2k_2}{b_2}}+\sqrt{-\frac{3k_2(-2b_2k_2+a_2)}{b_2}}\coth\left(\sqrt{-\frac{3k_2(-2b_2k_2+a_2)}{20b_2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)\right)^2}\right\}\times e^{i\left(-k_2x-\frac{k_2^2(119b_2^2k_2^2-119a_2b_2k_2+36a_2^2)}{25b_2}t+\zeta_2\right)} \tag{31}$$

with

$b_jk_j(-2b_jk_j+a_j)<0.$

Singular periodic solutions are

$$u(x,t)=\left\{SA_2-\frac{2A_2S\sqrt{\frac{6b_1k_1^2+20Sb_1-3a_1k_1}{b_1}}}{\sqrt{\frac{6b_1k_1^2+20Sb_1-3a_1k_1}{b_1}}-\sqrt{\frac{3k_1(-2b_1k_1+a_1)}{b_1}}\tan\left(\sqrt{\frac{3k_1(-2b_1k_1+a_1)}{20b_1}}(x-(-3a_1k_1^2+4b_1k_1^3)t)\right)}+\frac{20A_2S^2}{\left(\sqrt{\frac{6b_1k_1^2+20Sb_1-3a_1k_1}{b_1}}-\sqrt{\frac{3k_1(-2b_1k_1+a_1)}{b_1}}\tan\left(\sqrt{\frac{3k_1(-2b_1k_1+a_1)}{20b_1}}(x-(-3a_1k_1^2+4b_1k_1^3)t)\right)\right)^2}\right\}\times e^{i\left(-k_1x-\frac{k_1^2(119b_1^2k_1^2-119a_1b_1k_1+36a_1^2)}{25b_1}t+\zeta_1\right)}, \tag{32}$$

$$v(x,t)=\left\{SA_2-\frac{2A_2S\sqrt{\frac{6b_2k_2^2+20Sb_2-3a_2k_2}{b_2}}}{\sqrt{\frac{6b_2k_2^2+20Sb_2-3a_2k_2}{b_2}}-\sqrt{\frac{3k_2(-2b_2k_2+a_2)}{b_2}}\tan\left(\sqrt{\frac{3k_2(-2b_2k_2+a_2)}{20b_2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)}\right\}$$
$$+\frac{20A_2S^2}{\left(\sqrt{\frac{6b_2k_2^2+20Sb_2-3a_2k_2}{b_2}}-\sqrt{\frac{3k_2(-2b_2k_2+a_2)}{b_2}}\tan\left(\sqrt{\frac{3k_2(-2b_2k_2+a_2)}{20b_2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)\right)^2}$$
$$\times e^{i\left(-k_2x-\frac{k_2{}^2(119b_2^2k_2^2-119a_2b_2k_2+36a_2^2)}{25b_2}t+\zeta_2'\right)}, \tag{33}$$

$$u(x,t)=\left\{SA_2-\frac{2A_2S\sqrt{\frac{6b_1k_1^2+20Sb_1-3a_1k_1}{b_1}}}{\sqrt{\frac{6b_1k_1^2+20Sb_1-3a_1k_1}{b_1}}+\sqrt{\frac{3k_1(-2b_1k_1+a_1)}{b_1}}\cot\left(\sqrt{\frac{3k_1(-2b_1k_1+a_1)}{20b_1}}(x-(-3a_1k_1^2+4b_1k_1^3)t)\right)}\right\}$$
$$+\frac{20A_2S^2}{\left(\sqrt{\frac{6b_1k_1^2+20Sb_1-3a_1k_1}{b_1}}+\sqrt{\frac{3k_1(-2b_1k_1+a_1)}{b_1}}\cot\left(\sqrt{\frac{3k_1(-2b_1k_1+a_1)}{20b_1}}(x-(-3a_1k_1^2+4b_1k_1^3)t)\right)\right)^2}$$
$$\times e^{i\left(-k_1x-\frac{k_1{}^2(119b_1^2k_1^2-119a_1b_1k_1+36a_1^2)}{25b_1}t+\zeta_1'\right)}, \tag{34}$$

$$v(x,t)=\left\{SA_2-\frac{2A_2S\sqrt{\frac{6b_2k_2^2+20Sb_2-3a_2k_2}{b_2}}}{\sqrt{\frac{6b_2k_2^2+20Sb_2-3a_2k_2}{b_2}}+\sqrt{\frac{3k_2(-2b_2k_2+a_2)}{b_2}}\cot\left(\sqrt{\frac{3k_2(-2b_2k_2+a_2)}{20b_2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)}\right\}$$
$$+\frac{20A_2S^2}{\left(\sqrt{\frac{6b_2k_2^2+20Sb_2-3a_2k_2}{b_2}}+\sqrt{\frac{3k_2(-2b_2k_2+a_2)}{b_2}}\cot\left(\sqrt{\frac{3k_2(-2b_2k_2+a_2)}{20b_2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)\right)^2}$$
$$\times e^{i\left(-k_2x-\frac{k_2{}^2(119b_2^2k_2^2-119a_2b_2k_2+36a_2^2)}{25b_2}t+\zeta_2'\right)} \tag{35}$$

with

$$b_jk_j(-2b_jk_j+a_j)>0.$$

Parabolic law

The CQ-NLSE equation in polarization-preserving fibers with parabolic law is given by

$$iq_t+iaq_{xxx}+bq_{xxxx}+(c\,|q|^2+d\,|q|^4)q=0. \tag{36}$$

The constant a is the coefficient of the 3OD while the constant b is the coefficient of the 4OD. Also, the coefficients c, d provide the parabolic nonlinearity effect which has been quite popular recently and is known as a extended state of Kerr fiber nonlinearity.

The CQ-NLSE equation birefringent fibers with parabolic law is:

$$iu_t+ia_1u_{xxx}+b_1u_{xxxx}+(c_1\,|u|^2+d_1\,|v|^2)u+(e_1\,|u|^4+g_1\,|u|^2|v|^2+h_1\,|v|^4)u=0,$$
$$iv_t+ia_2v_{xxx}+b_2v_{xxxx}+(c_2\,|v|^2+d_2\,|u|^2)v+(e_2\,|v|^4+g_2\,|v|^2|u|^2+h_2\,|u|^4)v=0 \tag{37}$$

with constants c_j, e_j and d_j, g_j, h_j which represent SPM and XPM respectively. Also, the 4WM is discarded.

If Eq. (9) is inserted into Eq. (37), we get the equation

$$b_jU_j^{(4)}+(3a_jk_j-6b_jk_j^2)U_j''-(w_j+a_jk_j^3-b_jk_j^4)U_j+(c_j+d_j)U_j^3+(e_j+g_j+h_j)U_j^5=0 \tag{38}$$

which emerged from the real equation along with $U_{\bar{j}}=U_j$.

By the aid of balancing rule in Eq. (38), Eq. (4) is

$$U_j=A_0+A_1\exp(-V(\vartheta)). \tag{39}$$

Upon substituting Eqs. (5) and (39) into Eq. (38), we get the equations

$$-6RSA_1b_jk_j^2+3RSA_1a_jk_j-A_0w_j+A_0^3c_j+A_0^3d_j+A_0^5e_j+A_0^5g_j+A_0^5h_j+R^3SA_1b_j+8RS^2A_1b_j+A_0b_jk_j^4-A_0a_jk_j^3=0, \tag{40}$$

$$22R^2SA_1b_j-6R^2A_1b_jk_j^2+3R^2A_1a_jk_j-12SA_1b_jk_j^2+6SA_1a_jk_j+R^4A_1b_j+16S^2A_1b_j+A_1b_jk_j^4-A_1a_jk_j^3+3A_0^2A_1c_j+3A_0^2A_1d_j+5A_0^4A_1e_j+5A_0^4A_1g_j+5A_0^4A_1h_j-A_1w_j=0, \tag{41}$$

$$A_1^5e_j+A_1^5g_j+A_1^5h_j+24b_jA_1=0, \tag{42}$$

$$5A_0A_1^4e_j+5A_0A_1^4g_j+5A_0A_1^4h_j+60RA_1b_j=0, \tag{43}$$

$$10A_0^2A_1^3e_j+10A_0^2A_1^3g_j+10A_0^2A_1^3h_j+50R^2A_1b_j+A_1^3c_j+A_1^3d_j-12A_1b_jk_j^2+40SA_1b_j+6A_1a_jk_j=0, \tag{44}$$

$$10A_0^3A_1^2e_j+10A_0^3A_1^2g_j+10A_0^3A_1^2h_j+15R^3A_1b_j-18RA_1b_jk_j^2+60RSA_1b_j+9RA_1a_jk_j+3A_0A_1^2c_j+3A_0A_1^2d_j=0. \tag{45}$$

Thus, the important results are

$$R=2\frac{A_0}{A_1},\quad e_j=-\frac{A_1{}^4g_j+A_1^4h_j+24b_j}{A_1^4}, \tag{46}$$

$$c_j=-\frac{A_1{}^4d_j-12A_1^2b_jk_j^2+40SA_1^2b_j+6A_1^2a_jk_j-40A_0^2b_j}{A_1^4}, \tag{47}$$

$$w_j=\frac{A_1^4b_jk_j^4-12SA_1^4b_jk_j^2-A_1^4a_jk_j^3+16S^2A_1^4b_j+6SA_1^4a_jk_j+12A_0^2A_1^2b_jk_j^2-32SA_0^2A_1^2b_j-6A_0^2A_1^2a_jk_j+16A_0^4b_j}{A_1^4}, \tag{48}$$

$$\mu=R^2-4S=-\frac{4(SA_1^2-A_0^2)}{A_1^2} \tag{49}$$

which appear from Eqs. (40)–(45). Moreover, if Eqs. (46)–(49)with Eq.

(6) are substituted into Eq. (39), the singular solitons evolve as:

$$u(x,t)=\left\{A_0-\frac{SA_1^2}{A_0+\sqrt{A_0^2-SA_1^2}\tanh\left(\sqrt{-\frac{SA_1{}^2-A_0^2}{A_1^2}}(x-(-3a_1k_1^2+4b_1k_1^3)t)\right)}\right\}\times e^{i\left(-k_1x+\frac{A_1^4b_1k_1^4-12SA_1^4b_1k_1^2-A_1^4a_1k_1^3+16S^2A_1^4b_1+6SA_1^4a_1k_1+12A_0^2A_1^2b_1k_1^2-32SA_0^2A_1^2b_1-6A_0^2A_1^2a_1k_1+16A_0^4b_1}{A_1^4}t+\zeta_1\right)},\tag{50}$$

$$v(x,t)=\left\{A_0-\frac{SA_1^2}{A_0+\sqrt{A_0^2-SA_1^2}\tanh\left(\sqrt{-\frac{SA_1{}^2-A_0^2}{A_1^2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)}\right\}\times e^{i\left(-k_2x+\frac{A_1^4b_2k_2^4-12SA_1^4b_2k_2^2-A_1^4a_2k_2^3+16S^2A_1^4b_2+6SA_1^4a_2k_2+12A_0^2A_1^2b_2k_2^2-32SA_0^2A_1^2b_2-6A_0^2A_1^2a_2k_2+16A_0^4b_2}{A_1^4}t+\zeta_2\right)},\tag{51}$$

with

$SA_1^2-A_0^2<0.$

Dark solitons are

$$u(x,t)=\left\{A_0-\frac{SA_1^2}{A_0+\sqrt{A_0^2-SA_1^2}\coth\left(\sqrt{-\frac{SA_1{}^2-A_0^2}{A_1^2}}(x-(-3a_1k_1^2+4b_1k_1^3)t)\right)}\right\}\times e^{i\left(-k_1x+\frac{A_1^4b_1k_1^4-12SA_1^4b_1k_1^2-A_1^4a_1k_1^3+16S^2A_1^4b_1+6SA_1^4a_1k_1+12A_0^2A_1^2b_1k_1^2-32SA_0^2A_1^2b_1-6A_0^2A_1^2a_1k_1+16A_0^4b_1}{A_1^4}t+\zeta_1\right)},\tag{52}$$

$$v(x,t)=\left\{A_0-\frac{SA_1^2}{A_0+\sqrt{A_0^2-SA_1^2}\coth\left(\sqrt{-\frac{SA_1{}^2-A_0^2}{A_1^2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)}\right\}\times e^{i\left(-k_2x+\frac{A_1^4b_2k_2^4-12SA_1^4b_2k_2^2-A_1^4a_2k_2^3+16S^2A_1^4b_2+6SA_1^4a_2k_2+12A_0^2A_1^2b_2k_2^2-32SA_0^2A_1^2b_2-6A_0^2A_1^2a_2k_2+16A_0^4b_2}{A_1^4}t+\zeta_2\right)},\tag{53}$$

with

$SA_1^2-A_0^2<0.$

Singular periodic solutions are

$$u(x,t)=\left\{A_0-\frac{SA_1^2}{A_0-\sqrt{SA_1^2-A_0^2}\tan\left(\sqrt{\frac{SA_1{}^2-A_0^2}{A_1^2}}(x-(-3a_1k_1^2+4b_1k_1^3)t)\right)}\right\}\times e^{i\left(-k_1x+\frac{A_1^4b_1k_1^4-12SA_1^4b_1k_1^2-A_1^4a_1k_1^3+16S^2A_1^4b_1+6SA_1^4a_1k_1+12A_0^2A_1^2b_1k_1^2-32SA_0^2A_1^2b_1-6A_0^2A_1^2a_1k_1+16A_0^4b_1}{A_1^4}t+\zeta_1\right)},\tag{54}$$

$$v(x,t)=\left\{A_0-\frac{SA_1^2}{A_0-\sqrt{SA_1^2-A_0^2}\tan\left(\sqrt{\frac{SA_1{}^2-A_0^2}{A_1^2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)}\right\}\times e^{i\left(-k_2x+\frac{A_1^4b_2k_2^4-12SA_1^4b_2k_2^2-A_1^4a_2k_2^3+16S^2A_1^4b_2+6SA_1^4a_2k_2+12A_0^2A_1^2b_2k_2^2-32SA_0^2A_1^2b_2-6A_0^2A_1^2a_2k_2+16A_0^4b_2}{A_1^4}t+\zeta_2\right)},\tag{55}$$

$$u(x,t)=\left\{A_0-\frac{SA_1^2}{A_0+\sqrt{SA_1^2-A_0^2}\cot\left(\sqrt{\frac{SA_1{}^2-A_0^2}{A_1^2}}(x-(-3a_1k_1^2+4b_1k_1^3)t)\right)}\right\}\times e^{i\left(-k_1x+\frac{A_1^4b_1k_1^4-12SA_1^4b_1k_1^2-A_1^4a_1k_1^3+16S^2A_1^4b_1+6SA_1^4a_1k_1+12A_0^2A_1^2b_1k_1^2-32SA_0^2A_1^2b_1-6A_0^2A_1^2a_1k_1+16A_0^4b_1}{A_1^4}t+\zeta_1\right)},\tag{56}$$

$$v(x,t)=\left\{A_0-\frac{SA_1^2}{A_0+\sqrt{SA_1^2-A_0^2}\cot\left(\sqrt{\frac{SA_1{}^2-A_0^2}{A_1^2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)}\right\}\times e^{i\left(-k_2x+\frac{A_1^4b_2k_2^4-12SA_1^4b_2k_2^2-A_1^4a_2k_2^3+16S^2A_1^4b_2+6SA_1^4a_2k_2+12A_0^2A_1^2b_2k_2^2-32SA_0^2A_1^2b_2-6A_0^2A_1^2a_2k_2+16A_0^4b_2}{A_1^4}t+\zeta_2\right)},\tag{57}$$

with

$SA_1^2-A_0^2>0.$

Quadratic-cubic law

The CQ-NLSE equation in polarization-maintaining fibers with the quadratic-cubic law is given by

$$iq_t+iaq_{xxx}+bq_{xxxx}+(c\,|q|+d\,|q|^2)q=0.\tag{58}$$

The constant a is the coefficient of the 3OD while the constant b is the coefficient of the 4OD. Also, the coefficients c, d provide the quadratic-cubic law effect.

The CQ-NLSE equation in birefringent fibers with the quadratic-cubic law is:

$$iu_t+ia_1u_{xxx}+b_1u_{xxxx}+c_1u\sqrt{|u|^2+|v|^2+uv^*+u^*v}+(d_1\,|u|^2+e_1\,|v|^2)u=0,$$
$$iv_t+ia_2v_{xxx}+b_2v_{xxxx}+c_2v\sqrt{|v|^2+|u|^2+vu^*+v^*u}+(d_2\,|v|^2+e_2\,|u|^2)v=0\tag{59}$$

with the constants d_j and e_j which assure SPM and XPM respectively. Also, the constant c_j assures the existence of the quadratic media in birefringent fibers where the first two terms correspond to SPM and XPM sequentially while the last two terms refer to 4WM.

If Eq. (9) is inserted into Eq. (59), we arrive at

$$b_jU_j^{(4)}+(3a_jk-6b_jk^2)U_j''-(w+a_jk^3-b_jk^4)U_j+2c_jU_j^2+(d_j+e_j)U_j^3=0\tag{60}$$

which shows up from the real part equation along with

$$U_{\tilde{j}}=U_j,\tag{61}$$

$$\varphi(x,t)=-kx+wt+\zeta.\tag{62}$$

In consequence of using balancing rule in Eq. (60), Eq. (4) is given as

$$U_j=A_0+A_1\exp(-V(\vartheta))+A_2\exp(-2V(\vartheta)).\tag{63}$$

When Eqs. (5) and (63) are inserted into Eq. (60), one notices

$$-6RSk^2A_1b_j+3RSkA_1a_j+R^3SA_1b_j+14R^2S^2A_2b_j+8RS^2A_1b_j-12S^2k^2A_2b_j+6S^2kA_2a_j+16S^3A_2b_j+k^4A_0b_j-k^3A_0a_j-wA_0+2A_0^2c_j+A_0^3d_j+A_0^3e_j=0,\tag{64}$$

$$30R^3SA_2b_j + 22R^2SA_1b_j + 120RS^2A_2b_j - 6R^2k^2A_1b_j + 3R^2kA_1a_j - 12Sk^2A_1b_j + 6SkA_1a_j + R^4A_1b_j + 16S^2A_1b_j + k^4A_1b_j - k^3A_1a_j + 4A_0A_1c_j + 3A_0^2A_1d_j + 3A_0^2A_1e_j - wA_1 - 36RSk^2A_2b_j + 18RSkA_2a_j = 0, \tag{65}$$

$$232R^2SA_2b_j + 60RSA_1b_j - 24R^2k^2A_2b_j + 12R^2kA_2a_j - 18Rk^2A_1b_j - 48Sk^2A_2b_j + 9RkA_1a_j + 24SkA_2a_j + 16R^4A_2b_j + 15R^3A_1b_j + 136S^2A_2b_j + k^4A_2b_j - k^3A_2a_j + 4A_0A_2c_j + 3A_0^2A_2d_j + 3A_0^2A_2e_j + 3A_0A_1^2d_j + 3A_0A_1^2e_j - wA_2 + 2A_1^2c_j = 0, \tag{66}$$

$$A_2^3d_j + A_2^3e_j + 120A_2b_j,\ 3A_1A_2^2d_j + 3A_1A_2^2e_j + 336RA_2b_j + 24A_1b_j = 0, \tag{67}$$

$$330R^2A_2b_j - 36k^2A_2b_j + 3A_0A_2^2d_j + 3A_0A_2^2e_j + 3A_1^2A_2d_j + 3A_1^2A_2e_j + 60RA_1b_j + 240SA_2b_j + 18kA_2a_j + 2A_2^2c_j = 0, \tag{68}$$

$$130R^3A_2b_j - 60Rk^2A_2b_j + 50R^2A_1b_j + 440RSA_2b_j + 30RkA_2a_j - 12k^2A_1b_j + 6A_0A_1A_2d_j + 6A_0A_1A_2e_j + A_1^3d_j + A_1^3e_j + 40SA_1b_j + 6kA_1a_j + 4A_1A_2c_j = 0. \tag{69}$$

Thus, the important results are

$$R = \frac{A_1}{A_2},\quad S = \frac{A_0}{A_2},\quad d_j = -\frac{A_2{}^2e_j + 120b_j}{A_2^2}, \tag{70}$$

$$c_j = \frac{3(6k^2A_2^2b_j - 3kA_2^2a_j + 20A_0A_2b_j - 5A_1^2b_j)}{A_2^3}, \tag{71}$$

$$w = \frac{k^4A_2^4b_j - k^3A_2^4a_j + 24k^2A_0A_2^3b_j - 6k^2A_1^2A_2^2b_j - 12kA_0A_2^3a_j + 3kA_1^2A_2^2a_j + 16A_0^2A_2^2b_j - 8A_0A_1^2A_2b_j + A_1^4b_j}{A_2^4}, \tag{72}$$

$$\mu = R^2 - 4S = -\frac{4A_0A_2 - A_1^2}{A_2^2} \tag{73}$$

which comes from Eqs. (64)–(69) Moreover, if Eqs. (70)–(73)with Eq. (6) are substituted into Eq. (63), the singular solitons fall out as

$$u(x,t) = \left\{A_0 - \frac{2A_1A_0}{A_1 + \sqrt{A_1^2 - 4A_0A_2}\tanh\left(\sqrt{-\frac{4A_0A_2 - A_1^2}{4A_2^2}}(x - (-3a_1k_1^2 + 4b_1k_1^3)t)\right)} + \frac{4A_2A_0^2}{\left(A_1 + \sqrt{A_1^2 - 4A_0A_2}\tanh\left(\sqrt{-\frac{4A_0A_2 - A_1^2}{4A_2^2}}(x - (-3a_1k_1^2 + 4b_1k_1^3)t)\right)\right)^2}\right\} \times e^{i\left(-kx + \frac{k^4A_2^4b_1 - k^3A_2^4a_1 + 24k^2A_0A_2^3b_1 - 6k^2A_1^2A_2^2b_1 - 12kA_0A_2^3a_1 + 3kA_1^2A_2^2a_1 + 16A_0^2A_2^2b_1 - 8A_0A_1^2A_2b_1 + A_1^4b_1}{A_2^4}t + \zeta\right)}, \tag{74}$$

$$v(x,t) = \left\{A_0 - \frac{2A_1A_0}{A_1 + \sqrt{A_1^2 - 4A_0A_2}\tanh\left(\sqrt{-\frac{4A_0A_2 - A_1^2}{4A_2^2}}(x - (-3a_2k_2^2 + 4b_2k_2^3)t)\right)} + \frac{4A_2A_0^2}{\left(A_1 + \sqrt{A_1^2 - 4A_0A_2}\tanh\left(\sqrt{-\frac{4A_0A_2 - A_1^2}{4A_2^2}}(x - (-3a_2k_2^2 + 4b_2k_2^3)t)\right)\right)^2}\right\} \times e^{i\left(-kx + \frac{k^4A_2^4b_2 - k^3A_2^4a_2 + 24k^2A_0A_2^3b_2 - 6k^2A_1^2A_2^2b_2 - 12kA_0A_2^3a_2 + 3kA_1^2A_2^2a_2 + 16A_0^2A_2^2b_2 - 8A_0A_1^2A_2b_2 + A_1^4b_2}{A_2^4}t + \zeta\right)}, \tag{75}$$

with

$$4A_0A_2 - A_1^2 < 0.$$

Dark solitons are

$$u(x,t) = \left\{A_0 - \frac{2A_1A_0}{A_1 + \sqrt{A_1^2 - 4A_0A_2}\coth\left(\sqrt{-\frac{4A_0A_2 - A_1^2}{4A_2^2}}(x - (-3a_1k_1^2 + 4b_1k_1^3)t)\right)} + \frac{4A_2A_0^2}{\left(A_1 + \sqrt{A_1^2 - 4A_0A_2}\coth\left(\sqrt{-\frac{4A_0A_2 - A_1^2}{4A_2^2}}(x - (-3a_1k_1^2 + 4b_1k_1^3)t)\right)\right)^2}\right\} \times e^{i\left(-kx + \frac{k^4A_2^4b_1 - k^3A_2^4a_1 + 24k^2A_0A_2^3b_1 - 6k^2A_1^2A_2^2b_1 - 12kA_0A_2^3a_1 + 3kA_1^2A_2^2a_1 + 16A_0^2A_2^2b_1 - 8A_0A_1^2A_2b_1 + A_1^4b_1}{A_2^4}t + \zeta\right)}, \tag{76}$$

$$v(x,t) = \left\{A_0 - \frac{2A_1A_0}{A_1 + \sqrt{A_1^2 - 4A_0A_2}\coth\left(\sqrt{-\frac{4A_0A_2 - A_1^2}{4A_2^2}}(x - (-3a_2k_2^2 + 4b_2k_2^3)t)\right)} + \frac{4A_2A_0^2}{\left(A_1 + \sqrt{A_1^2 - 4A_0A_2}\coth\left(\sqrt{-\frac{4A_0A_2 - A_1^2}{4A_2^2}}(x - (-3a_2k_2^2 + 4b_2k_2^3)t)\right)\right)^2}\right\} \times e^{i\left(-kx + \frac{k^4A_2^4b_2 - k^3A_2^4a_2 + 24k^2A_0A_2^3b_2 - 6k^2A_1^2A_2^2b_2 - 12kA_0A_2^3a_2 + 3kA_1^2A_2^2a_2 + 16A_0^2A_2^2b_2 - 8A_0A_1^2A_2b_2 + A_1^4b_2}{A_2^4}t + \zeta\right)}, \tag{77}$$

with

$$4A_0A_2 - A_1^2 < 0.$$

Singular periodic solutions are

$$u(x,t) = \left\{A_0 - \frac{2A_1A_0}{A_1 - \sqrt{4A_0A_2 - A_1^2}\tan\left(\sqrt{\frac{4A_0A_2 - A_1^2}{4A_2^2}}(x - (-3a_1k_1^2 + 4b_1k_1^3)t)\right)} + \frac{4A_2A_0^2}{\left(A_1 - \sqrt{4A_0A_2 - A_1^2}\tan\left(\sqrt{\frac{4A_0A_2 - A_1^2}{4A_2^2}}(x - (-3a_1k_1^2 + 4b_1k_1^3)t)\right)\right)^2}\right\} \times e^{i\left(-kx + \frac{k^4A_2^4b_1 - k^3A_2^4a_1 + 24k^2A_0A_2^3b_1 - 6k^2A_1^2A_2^2b_1 - 12kA_0A_2^3a_1 + 3kA_1^2A_2^2a_1 + 16A_0^2A_2^2b_1 - 8A_0A_1^2A_2b_1 + A_1^4b_1}{A_2^4}t + \zeta\right)}, \tag{78}$$

$$v(x,t) = \left\{A_0 - \frac{2A_1A_0}{A_1 - \sqrt{4A_0A_2 - A_1^2}\tan\left(\sqrt{\frac{4A_0A_2 - A_1^2}{4A_2^2}}(x - (-3a_2k_2^2 + 4b_2k_2^3)t)\right)} + \frac{4A_2A_0^2}{\left(A_1 - \sqrt{4A_0A_2 - A_1^2}\tan\left(\sqrt{\frac{4A_0A_2 - A_1^2}{4A_2^2}}(x - (-3a_2k_2^2 + 4b_2k_2^3)t)\right)\right)^2}\right\} \times e^{i\left(-kx + \frac{k^4A_2^4b_2 - k^3A_2^4a_2 + 24k^2A_0A_2^3b_2 - 6k^2A_1^2A_2^2b_2 - 12kA_0A_2^3a_2 + 3kA_1^2A_2^2a_2 + 16A_0^2A_2^2b_2 - 8A_0A_1^2A_2b_2 + A_1^4b_2}{A_2^4}t + \zeta\right)}, \tag{79}$$

$$u(x,t) = \left\{A_0 - \frac{2A_1A_0}{A_1 + \sqrt{4A_0A_2 - A_1^2}\cot\left(\sqrt{\frac{4A_0A_2 - A_1^2}{4A_2^2}}(x - (-3a_1k_1^2 + 4b_1k_1^3)t)\right)} + \frac{4A_2A_0^2}{\left(A_1 + \sqrt{4A_0A_2 - A_1^2}\cot\left(\sqrt{\frac{4A_0A_2 - A_1^2}{4A_2^2}}(x - (-3a_1k_1^2 + 4b_1k_1^3)t)\right)\right)^2}\right\} \times e^{i\left(-kx + \frac{k^4A_2^4b_1 - k^3A_2^4a_1 + 24k^2A_0A_2^3b_1 - 6k^2A_1^2A_2^2b_1 - 12kA_0A_2^3a_1 + 3kA_1^2A_2^2a_1 + 16A_0^2A_2^2b_1 - 8A_0A_1^2A_2b_1 + A_1^4b_1}{A_2^4}t + \zeta\right)}, \tag{80}$$

$$v(x,t)=\left\{A_0-\frac{2A_1A_0}{A_1+\sqrt{4A_0A_2-A_1^2}\cot\left(\sqrt{\frac{4A_0A_2-A_1^2}{4A_2^2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)}+\frac{4A_2A_0^2}{\left(A_1+\sqrt{4A_0A_2-A_1^2}\cot\left(\sqrt{\frac{4A_0A_2-A_1^2}{4A_2^2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)\right)^2}\right\}\times e^{i\left(-kx+\frac{k^4A_2^4b_2-k^3A_2^4a_2+24k^2A_0A_2^3b_2-6k^2A_1^2A_2^2b_2-12kA_0A_2^3a_2+3kA_1^2A_2^2a_2+16A_0^2A_2^2b_2-8A_0A_1^2A_2b_2+A_1^4b_2}{A_2^4}t+\zeta\right)} \quad (81)$$

with

$$4A_0A_2-A_1^2>0.$$

Non-local law

The CQ-NLSE equation in polarization-preserving fibers with the non-local law is given by

$$iq_t+iaq_{xxx}+bq_{xxxx}+c(|q|^2)_{xx}q=0. \quad (82)$$

The constant a is the coefficient of the 3OD while the constants b is the coefficient of the 4OD. Also, the coefficient c provides the non-local law effect.

The CQ-NLSE equation in birefringent fibers with the non-local nonliearity is:

$$iu_t+ia_1u_{xxx}+b_1u_{xxxx}+(c_1(|u|^2)_{xx}+d_1(|v|^2)_{xx})u=0,$$
$$iv_t+ia_2v_{xxx}+b_2v_{xxxx}+(c_2(|v|^2)_{xx}+d_2(|u|^2)_{xx})v=0 \quad (83)$$

with the constants c_j and d_j which are SPM and XPM.

If Eq. (9) is inserted into Eq. (83), we get the equation

$$b_jU_j^{(4)}+(3a_jk_j-6b_jk_j^2)U_j''-(w_j+a_jk_j^3-b_jk_j^4)U_j+2(c_j+d_j)U_j(U_j')^2+2(c_j+d_j)U_j^2U_j''=0 \quad (84)$$

which comes out from real part equation along with $U_{\tilde{j}}=U_j$.

From balancing rule in Eq. (84), Eq. (4) turns out to be

$$U_j=A_0+A_1\exp(-V(\vartheta)). \quad (85)$$

As long as Eq. (5) with Eq. (85) are inserted into Eq. (84), we get the equations

$$2RSA_0^2A_1c_j+2RSA_0^2A_1d_j-6RSA_1b_jk_j^2+3RSA_1a_jk_j-A_0w_j+R^3SA_1b_j+8RS^2A_1b_j+2S^2A_0A_1^2c_j+2S^2A_0A_1^2d_j+A_0b_jk_j^4-A_0a_jk_j^3=0, \quad (86)$$

$$22R^2SA_1b_j-6R^2A_1b_jk_j^2+3R^2A_1a_jk_j-12SA_1b_jk_j^2+6SA_1a_jk_j+2R^2A_0^2A_1c_j+2R^2A_0^2A_1d_j+4SA_0^2A_1c_j+4SA_0^2A_1d_j+R^4A_1b_j+16S^2A_1b_j+A_1b_jk_j^4-A_1a_jk_j^3+2S^2A_1^3c_j+2S^2A_1^3d_j-A_1w_j+8RSA_0A_1^2d_j+8RSA_0A_1^2c_j=0, \quad (87)$$

$$6A_1^3c_j+6A_1^3d_j+24b_jA_1=0, \quad (88)$$

$$10RA_1^3c_j+10RA_1^3d_j+10A_0A_1^2c_j+10A_0A_1^2d_j+60RA_1b_j=0, \quad (89)$$

$$4R^2A_1^3c_j+4R^2A_1^3d_j+16RA_0A_1^2c_j+16RA_0A_1^2d_j+8SA_1^3c_j+8SA_1^3d_j+50R^2A_1b_j+4A_0^2A_1c_j+4A_0^2A_1d_j-12A_1b_jk_j^2+40SA_1b_j+6A_1a_jk_j=0, \quad (90)$$

$$6R^2A_0A_1^2c_j+6R^2A_0A_1^2d_j+6RSA_1^3c_j+6RSA_1^3d_j+15R^3A_1b_j+6RA_0^2A_1c_j+6RA_0^2A_1d_j-18RA_1b_jk_j^2+12SA_0A_1^2c_j+12SA_0A_1^2d_j+60RSA_1b_j+9RA_1a_jk_j=0. \quad (91)$$

Thus, the important results are

$$S=\frac{R^2b_j+6b_jk_j^2-3a_jk_j}{4b_j},\quad A_0=\frac{A_1R}{2}, \quad (92)$$

$$c_j=-\frac{A_1{}^2d_j+4b_j}{A_1^2},\quad w_j=-k_j^3(-b_jk_j+a_j), \quad (93)$$

$$\mu=R^2-4S=\frac{3k_j(-2b_jk_j+a_j)}{b_j} \quad (94)$$

which are emerged from Eqs. (86)–(91). Moreover, if Eqs. (92)–(94) with Eq. (6) are substituted into Eq. (85), the singular solitons are

$$u(x,t)=\left\{SA_2-\frac{A_1(R^2b_1+6b_1k_1^2-3a_1k_1)}{2Rb_1+\sqrt{12b_1k_1(-2b_1k_1+a_1)}\tanh\left(\sqrt{\frac{3k_1(-2b_1k_1+a_1)}{4b_1}}(x-(-3a_1k_1^2+4b_1k_1^3)t)\right)}\right\}\times e^{i(-k_1x-k_1^3(-b_1k_1+a_1)t+\zeta_1)}, \quad (95)$$

$$v(x,t)=\left\{SA_2-\frac{A_1(R^2b_2+6b_2k_2^2-3a_2k_2)}{2Rb_2+\sqrt{12b_2k_2(-2b_2k_2+a_2)}\tanh\left(\sqrt{\frac{3k_2(-2b_2k_2+a_2)}{4b_2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)}\right\}\times e^{i(-k_2x-k_2^3(-b_2k_2+a_2)t+\zeta_2)} \quad (96)$$

with

$$b_jk_j(-2b_jk_j+a_j)>0.$$

Dark solitons are

$$u(x,t)=\left\{SA_2-\frac{A_1(R^2b_1+6b_1k_1^2-3a_1k_1)}{2Rb_1+\sqrt{12b_1k_1(-2b_1k_1+a_1)}\coth\left(\sqrt{\frac{3k_1(-2b_1k_1+a_1)}{4b_1}}(x-(-3a_1k_1^2+4b_1k_1^3)t)\right)}\right\}\times e^{i(-k_1x-k_1^3(-b_1k_1+a_1)t+\zeta_1)}, \quad (97)$$

$$v(x,t)=\left\{SA_2-\frac{A_1(R^2b_2+6b_2k_2^2-3a_2k_2)}{2Rb_2+\sqrt{12b_2k_2(-2b_2k_2+a_2)}\coth\left(\sqrt{\frac{3k_2(-2b_2k_2+a_2)}{4b_2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)}\right\}\times e^{i(-k_2x-k_2^3(-b_2k_2+a_2)t+\zeta_2)} \quad (98)$$

with

$$b_jk_j(-2b_jk_j+a_j)>0.$$

Singular periodic solutions are

$$u(x,t)=\left\{SA_2-\frac{A_1(R^2b_1+6b_1k_1^2-3a_1k_1)}{2Rb_1-\sqrt{12b_1k_1(2b_1k_1-a_1)}\tan\left(\sqrt{-\frac{3k_1(-2b_1k_1+a_1)}{4b_1}}(x-(-3a_1k_1^2+4b_1k_1^3)t)\right)}\right\}\times e^{i(-k_1x-k_1^3(-b_1k_1+a_1)t+\zeta_1)}, \quad (99)$$

$$v(x,t)=\left\{SA_2-\frac{A_1(R^2b_2+6b_2k_2^2-3a_2k_2)}{2Rb_2-\sqrt{12b_2k_2(2b_2k_2-a_2)}\tan\left(\sqrt{-\frac{3k_2(-2b_2k_2+a_2)}{4b_2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)}\right\}\times e^{i(-k_2x-k_2^3(-b_2k_2+a_2)t+\zeta_2)}, \quad (100)$$

$$u(x,t)=\left\{SA_2-\frac{A_1(R^2b_1+6b_1k_1^2-3a_1k_1)}{2Rb_1+\sqrt{12b_1k_1(2b_1k_1-a_1)}\tan\left(\sqrt{-\frac{3k_1(-2b_1k_1+a_1)}{4b_1}}(x-(-3a_1k_1^2+4b_1k_1^3)t)\right)}\right\}\times e^{i(-k_1x-k_1^3(-b_1k_1+a_1)t+\zeta_1)}, \quad (101)$$

$$v(x,t)=\left\{SA_2-\frac{A_1(R^2b_2+6b_2k_2^2-3a_2k_2)}{2Rb_2+\sqrt{12b_2k_2(2b_2k_2-a_2)}\tan\left(\sqrt{-\frac{3k_2(-2b_2k_2+a_2)}{4b_2}}(x-(-3a_2k_2^2+4b_2k_2^3)t)\right)}\right\}\times e^{i(-k_2x-k_2^3(-b_2k_2+a_2)t+\zeta_2)} \quad (102)$$

with

$$b_jk_j(-2b_jk_j+a_j)<0.$$

Conclusions

The current work is about the extraction of CQ dark and singular solitons in birefringent fibers that is studied with four forms of nonlinear refractive index. The exp-function expansion is the integration algorithm that has been implemented successfully, in this context. The results are a sequel to the ones that have been reported earlier [19]. Later, this method can be implemented into several additional forms of nonlinearities by the aid of more powerful integration algorithms [10–18,20]. One would also need to set up the model for CQ solitons, having such nonlinearities, with DWDM topology. These results are currently pending. Additionally, the model also needs to be studied in birefringent fibers using numerical algorithms such as Laplace-Adomian decomposition scheme, variational iteration method and many others [8,16]. The results of those research are all under way. They will be made available with time.

Acknowledgments

The research work of fifth author (QZ) was supported by the National Natural Science Foundation of China (Grant Nos. 11705130 and 11547149); this author was also sponsored by the Chutian Scholar Program of Hubei Government in China. The research work of the eighth author (MRB) was supported by the grant NPRP 11S-1126-170033 from QNRF and he is thankful for it.

The authors also declare that there is no conflict of interest.

References

[1] Bansal A, Biswas A, Zhou Q, Babatin MM. Lie symmetry analysis for cubic-quartic nonlinear Schrödinger's equation. Optik 2018;169:12–5.
[2] Biswas A, Triki H, Zhou Q, Moshokoa SP, Ullah MZ, Belic M. Cubic–quartic optical solitons in Kerr and power–law media. Optik 2017;144:357–62.
[3] Biswas A, Kara AH, Ullah MZ, Zhou Q, Triki H, Belic M. Conservation laws for cubic–quartic optical solitons in Kerr and power–law media. Optik 2017;145:650–4.
[4] Biswas A, Arshed S. Application of semi inverse variational principle to cubic–quartic optical solitons having Kerr and power law nonlinearity. Optik 2018;172:847–50.
[5] Blanco-Redondo A, Sterke CMD, Sipe JE, Krauss TF, Eggleton BJ, Husko C. Pure–quartic solitons. Nat Commun 2016;7:10427.
[6] Blanco A, Redondo, Sterke CMD, Sipe JE, Krauss TF, Eggleton BJ, Husko C. Erratum: pure–quartic solitons. Nat Commun 2016;7:11048.
[7] Das A, Biswas A, Ekici M, Khan S, Zhou Q, Moshokoa SP. Suppressing internet bottleneck with fractional temporal evolution of cubic quartic optical solitons. Optik 2019;182:303–7.
[8] Gonzalez-Gaxiola O, Biswas A, Mallawi F, Belic MR. Cubic-quartic bright optical solitons with improved Adomian decomposition method. J Adv Res 2020;21:161–7.
[9] Kohl RW, Biswas A, Ekici M, Zhou Q, Moshokoa SP, Belic MR. Cubic–quartic optical soliton perturbation by semi–inverse variational principle. Optik 2019;185:45–9.
[10] Kudryashov NA. A generalized model for description of propagation pulses in optical fiber. Optik 2019;189:42–52.
[11] Kyudryashov NA. First integrals and general solution of the traveling wave reduction for Schrödinger equation with anti–cubic nonlinearity. Optik 2019;185:665–71.
[12] Kudryashov NA. Traveling wave solutions of the generalized nonlinear Schrödinger equation with cubic–quintic nonlinearity. Optik 2019;188:27–35.
[13] Kudryashov NA. The PainlevÃ approach for finding solitary wave solutions of nonlinear nonintegrable differential equations. Optik 2019;183:642–9.
[14] Kudryashov NA. Solitary and periodic waves of the hierarchy for propagation pulse in optical fiber. Optik 2019;194:163060.
[15] Kudryashov NA. Construction of nonlinear differential equations for description of propagation pulses in optical fiber. Optik 2019;192:162964.
[16] Wazwaz AM. A variety of optical solitons for nonlinear Schrödinger equation with detuning term by the variational iteration method. Optik 2019;196:163169.
[17] Wazwaz AM. Bright and dark optical solitons for (2 + 1)–dimensional Schrödinger (NLS) equations in the anomalous dispersion regimes and the normal dispersive regimes. Optik 2019;192:162948.
[18] Xu G. Extended auxiliary equation method and its applications to three generalized NLS equations. Abstract Appl Anal 2014;2014:541370.
[19] Yildirim Y, Biswas A, Guggilla P, Mallawi F, Belic MR. Cubic–quartic optical solitons in birefringent fibers with four forms of nonlinear refractive index. To appear in Optik.
[20] Zayed EME, Alurrfi KAE. Solitons and other solutions for two nonlinear Schrodinger equations using the new mapping method. Optik 2017;144:132–48.
[21] Triki H, Biswas A, Zhou Q, Liu W, Ekici M, Alshomrani AS, Belic MR. Propagation of chirped optical similaritons in inhomogeneous tapered centrosymmetric nonlinear waveguides doped with resonant impurities. Laser Phys 2019;29(8):085401.
[22] Yan Y, Liu W. Stable transmission of solitons in the complex cubic–quintic Ginzburg-Landau equation with nonlinear gain and higher-order effects. Appl Math Lett 2019;98:171–6.
[23] Liu W, Zhang Y, Wazwaz AM, Zhou Q. Analytic study on triple–S, triple–triangle structure interactions for solitons in inhomogeneous multi–mode fiber. Appl Math Comput 2019;361:325–31.
[24] Guan X, Liu W, Zhou Q, Biswas A. Some lump solutions for a generalized (3 + 1)–dimensional Kadomtsev-Petviashvili equation. Appl Math Comput 2020;366:124757.
[25] Guan X, Zhou Q, Liu W. Lump and lump strip solutions to the (3 + 1)–dimensional generalized Kadomtsev-Petviashvili equation. Eur Phys J Plus 2019;134(7):371.