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On the application of systems of functions of special kind in mathematical physics

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Abstract. In the article considers the application of functional sequences of special kind for study of some boundary value problems of equations of mathematical physics. This sequences may be used in modelling of physical processes.

1. Introduction

Consider of functional sequences of special kind

$$\left\{ Re[W(x)]^n;\ Im[W(x)]^n \right\}_{n=0}^{\infty}, \tag{1}$$

where $W(x)$ continuous complex-valued function of $x \in [a,b]$. In [1-2], the basic properties such sequences are proved. In [3] Minzoni A.A.proved completeness in $L_2[0;2\pi]$ system of functions of special kind

$$\left\{ e^{-np(x)}\cos(nt);\ e^{-np(x)}\sin(nt) \right\}_{n=0}^{\infty} \tag{2}$$

where $p(x)$ real function $p(x) \in C^{1,\alpha}$. The proof is based on the fact that this sequence is applied to the solution of the following boundary value problem of (see [4])

$$\triangle w(x,y) = 0, \tag{3}$$

$$w(x,p(x)) = f(x)$$

$$w(0,y) = w(2\pi,y)$$

$$w(x,y) - boundary,$$

as $y \to +\infty$. The solution of this problem is given in the form of a series

$$w(x,y) = \sum_{n=0}^{\infty}(A_n e^{-ny}\cos(nx) + B_n e^{-ny}\sin(nx)),$$

where the coefficients satisfy the relation

$$f(x) = w(x, p(x)) = \sum_{n=0}^{\infty}(A_n e^{-np(x)}\cos(nx) + B_n e^{-np(x)}\sin(nx)) \qquad (4)$$

Moreover, the function $p(x)$ is subject to rather rigid conditions $p(x) \in C^{1,\alpha}[0; 2\pi]$. In fact, the sequence (2)(see Minzoni[3]) is a system of the form (1) for
$W(x) = e^{-p(x)+ix}$, i.e. $\{e^{-np(x)}\cos(nx); e^{-np(x)}\sin(nx)\}_{n=0}^{\infty}$, $x \in [0; 2\pi]$

2. Conditions of completeness of functional sequences in this kind in different function spaces

In fact, this result follows from the works of [1],[2], proved in a different way earlier than [3] and under more general conditions for the $p(x)$ function. The completeness of the system of functions (1) in different spaces is proved in [1], and the minimality in [2]. In [2], the minimality is proved and the sequence biorthogonally conjugate to (1) is given explicitly. The image of the segment $[0; 2\pi]$ when $W(x) = e^{-p(x)+ix}$, $x \in [0; 2\pi]$ displayed is the curve $\Gamma = W[0; 2\pi]$. In article [2] it is shown how the values of the interior angles at the corner points of the closed contour $\Gamma = W[0; 2\pi]$ are influenced by the "p" spaces $L_p[0; 2\pi]$, in which the sequence (2) is complete and minimal. In the simulation, the case when Γ is a smooth contour is most often used, because useful

Theorem 1. *Let $p(x)$ be a smooth real function, then*

1) if $p(0) = p(2\pi)$, then the sequence (2) is complete and minimal in $L_2[0; 2\pi]$

2) if $p(0) \neq p(2\pi)$, then the sequence (2) is densely complete in $L_2[0; 2\pi]$

Note that the sequence is densely complete in the corresponding space , which means that it remains complete in this space after removing any finite number of its elements.

If $p(0) \neq p(2\pi)$,then the contour Γ ($\Gamma = W[0; 2\pi]$) is open and the sequence (2) is densely complete in in any space $L_p[0; 2\pi]$, $p > 1$. If $p(0) = p(2\pi)$, then the contour Γ is closed, and the point 0 (origin) lies in the finite region bounded by this contour Γ, then this sequence (2) is complete and minimal in various functional spaces depending on the values of the angles at the angular points of the contour Γ (see [2]). At least for a smooth closed contour $\Gamma = W[0; 2\pi]$ the sequence (2) is complete and minimal in any space $L_p[0; 2\pi]$, $p > 1$.

Hence, the statement of Minzoni ([3], 1986) is a special case of the results ([1], 1977), ([2], 1981).

The following properties of the boundary value problem (3) of differential equations of mathematical physics are shown.

Boundary condition $p(0) \neq p(2\pi)$ of the problem (3) means the dense completeness of the sequence (2)in $L_2[0; 2\pi]$. Boundary condition $p(0) = p(2\pi)$ of the problem (3) means the completeness and minimality of the sequence (2)in $L_2[0; 2\pi]$.

Moreover, in this case, if equality (4) holds, then the article offers an effective way of calculating the coefficients A_n, B_n of this series. This method is analogous to the method for calculating the coefficients of a Fourier series in the classical trigonometric system. Namely, for the biorthogonal system to the sequence (2) the system $\{\varphi_n(t); \psi_n(t)\}$, $n = 0, 1, ...$

$\varphi_0(t) = \frac{1}{2\pi i}\frac{\chi\prime[W(t)]W\prime[(t)]}{\chi[W(t)]}$

$\varphi_n(t) = \frac{\chi\prime[W(t)]W\prime(t)}{\chi[W(t)]}\sum_1^n[C_k^{(n)}\chi^k(W(t)) - \overline{C}_k^{(n)}\chi^{-k}(W(t))]$,

$\psi_n(t) = -i\frac{\chi\prime[W(t)]W\prime(t)}{\chi[W(t)]}\sum_1^n[C_k^{(n)}\chi^k(W(t)) + \overline{C}_k^{(n)}\chi^{-k}(W(t))],$

where the function $\chi(w)$ conform maps the domain $Int\Gamma$ onto the unit disc $U = \{\zeta \in C \ |\zeta| < 1\}$, where $\chi(0) = 0$, $\chi\prime(0) > 0$ and $\chi\prime(W(t))$ is the angular boundary value of the derivative $\chi\prime(w))$ at the point $w = W(t)$.

The sequence of finite sets of $\{C_k^{(n)}\}_{k=1}^n$, $n = 0, 1, 2, ...$ complex constants and the form of the biorthogonal system are given in [2]. Then the coefficients A_n, B_n of the series (by the usual Fourier series formulas $A_n = \int_a^b f(t)\varphi_n(t)dt$, $B_n = \int_a^b f(t)\psi_n dt)$ are calculated.

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