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Analytical equation for the damping coefficient of the acoustic wave in the strong centrifugal field

S.V. Bogovalov, V.A. Kislov, I.V. Tronin

National Research Nuclear University “MEPhI” Kashirskoye shosse, 31, Moscow, 115409, Russia

E-mail: IVTronin@mephi.ru

Abstract. In the fast rotating gas (for example, in the gas centrifuge) three families of linear waves exist with different polarizations and law of dispersion. Analytical equation for the damping coefficient of the wave with the weakest damping is determined. This wave propagates strictly along the axis of rotation and the energy density of this wave concentrates at the wall of the rotor.

1. Introduction

Wave dynamics in rotating liquids and gases attracts attention for a long time. Although the waves in the rotating liquids were studied in many works [1, 2, 3], the properties of the waves in the gases were studied only in few works and the investigations have been limited by rather moderate rotational velocities [4]. Properties of these waves are interesting not only from the theoretical point of view rather than for practical use, in particular in physics of gas centrifuges (GC).

GC are explored for uranium isotope production starting with 50th years of the last century. In spite of this, the physics of the gas flow in the GC is still not fully understood. In our previous paper [5] we argued that the scoops used for the expulsion of the gas from the GC produce waves which propagate along the rotor. The properties of the waves in ideal gas compressed in the centrifugal field of the order 10^6 g (g – acceleration of gravity at the Earth surface) are rather specific.

The velocities of the conventional GC is about 600 – 700 m/s at the rotor radius about 6 – 9 cm [6]. The centrifugal acceleration is about 10^5 – 10^6 g at these parameters. Therefore, pressure of the working gas changes on 3-4 orders of magnitude at 1 cm radius variation. In these conditions conventional acoustic waves split into three families having different dispersion and polarization [5].

The waves in the gas are able to produce so-called “acoustic flow” which has been discussed starting with the work by Lord Rayleigh [7, 8]. The waves produced by the scoops are certainly able to produce an additional axial circulation in the gas centrifuge. Recently this was shown in direct numerical experiments [9]. Moreover, the numerical experiments [10] have shown that the waves change the gas flux from the waste chamber of GC provided that the waves can reach the opposite end of the rotor and product baffle. For the gas centrifuges this may result into reduction of the gas content in the gas centrifuges.



The waves with the weakest damping are especially interesting for us because they can strongly modify the impact of the scoops on the axial circulation and gas content in the GC. In this paper we focus our attention on these waves. They have dispersion law similar to the dispersion law of the conventional acoustic waves and are polarized along the rotational axis.

2. Analytical estimation of the damping

According to [11], the damping coefficient γ_d of a linear wave can be obtained as

$$\gamma_d = \frac{|\dot{E}_{mec}|}{2Ec}, \quad (1)$$

where \dot{E}_{mec} – rate of the mechanical energy dissipation, E – average energy of the wave. The rate of the mechanical energy dissipation equals to [11]

$$\dot{E}_{mec} = -\frac{\lambda}{T} \int (\nabla T)^2 dV - \frac{\eta}{2} \int \left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3} \delta_{ik} \frac{\partial v_l}{\partial x_l} \right)^2 dV, \quad (2)$$

and

$$E = \int \left(\frac{\rho_0 \bar{v}^2}{2} + \frac{c^2 \bar{\rho}^2}{2\rho_0} \right) dV. \quad (3)$$

Energy dissipation (2) is determined by velocity and temperature gradients. Average energy in the wave can be calculated by substitution of the axial velocity profile for dissipationless rotating gas (equation (54) from the work [5]) into integral (3)

$$v_z = v_0 \exp\left(\frac{(1-\gamma)\omega^2(r^2 - a^2)}{2c^2}\right) e^{i\Omega t}, \quad (4)$$

where $v_0 = \frac{c \bar{p}_w}{\gamma p_w}$ – amplitude of velocity in the wave, and γ is the adiabatic index of the gas. Initial density profile has a form

$$\rho_0 = \rho_w \exp\left(\frac{\gamma\omega^2(r^2 - a^2)}{2c^2}\right). \quad (5)$$

Then we obtain

$$E = 2\pi L \int_0^a \frac{\rho_w e^{A\left(\frac{r^2}{a^2}-1\right)} v^2}{2} r dr = \frac{E_0}{A} \frac{\gamma}{2-\gamma} \left(1 - \exp\left(-A\frac{(2-\gamma)}{\gamma}\right) \right), \quad (6)$$

where $A = \frac{M\omega^2 a^2}{2RT_0}$, E_0 – energy of the wave with amplitude v_0 in uniform gas with density ρ_w .

Calculation of integral (2) can be divided on two parts corresponding to the volume and surface contribution in to the damping as follows

$$\dot{E}_{mec} = \int_{r < a-h} (...) dV + \int_{a-h < r < a} (...) dV, \quad (7)$$

where h is the thickness of layer where the amplitude of the plane wave decays at approaching to the wall due to friction and heat exchange with the wall. The characteristic thickness of this layer is [11]:

$$h \approx \sqrt{\frac{2\nu}{\Omega}}, \quad (8)$$

where ν – kinematic viscosity, Ω – wave frequency.

2.1. Surface damping

In the region $a - h < r < a$ the perturbation of velocity and temperature go to zero due to friction and heat exchange with the wall. At the condition

$$h = \sqrt{\frac{2\nu}{\Omega}} \ll \frac{a}{A}. \quad (9)$$

the thickness of the surface layer is much less the characteristic scale of variation of density and pressure in the gas. In this case we can neglect variation of these values at the calculation of the surface integral in eq. (7). Then, this integral \dot{E}_{mec}^{surf} exactly coincides with the similar integral for an ordinary plane wave interacting with the wall and the damping coefficient equals

$$\gamma_d^{surf} = \frac{|\dot{E}_{mech}^{surf}|}{2Ec} = A \frac{(2 - \gamma)}{\gamma} \frac{|\dot{E}_{mec}^{surf}|}{2E_0c} \left(1 - \exp\left(-A \frac{(2 - \gamma)}{\gamma}\right) \right). \quad (10)$$

But

$$\frac{|\dot{E}_{mec}^{surf}|}{2E_0c} = \gamma_0^{surf}, \quad (11)$$

where γ_0^{surf} is the surface damping coefficient for the wave in the quiescent rotor (tube) given in [11]. Therefore, the surface damping in the rotating gas equals

$$\gamma_d^{surf} = \gamma_0^{surf} A \frac{(2 - \gamma)}{\gamma} \left(1 - \exp\left(-A \frac{(2 - \gamma)}{\gamma}\right) \right). \quad (12)$$

2.2. Volume damping

For the calculation of \dot{E}_{mec} in the volume it is necessary to have correct solution for the velocity and temperature variation in the wave. Exploration of the solution obtained in the dissipationless approximation gives strongly overestimated damping. Unfortunately, we have no analytical solution of the problem. Therefore, we give approximate estimate of the volume damping. Integral in (2) is accumulated in all the volume of the rotor. However, viscosity and thermal conductivity strongly suppress the wave reducing the its amplitude to zero below radius a_{min} where the viscous terms in the Navier-Stocks equations dominate over the inertial terms. Equation for the axial velocity v_z of the rotating gas in the cylindrical coordinate system in the axisymmetric approximation has the form [11]:

$$\rho \frac{\partial v_z}{\partial t} + \rho \left(v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \eta \left(\Delta v_z + \frac{1}{3} \frac{\partial \operatorname{div} \vec{v}}{\partial z} \right). \quad (13)$$

The viscous terms in eq. (13) dominate over the inertial terms when

$$\rho c < \eta k, \quad (14)$$

where c is the conventional sound velocity, η dynamical viscosity of the gas and k is the wave vector of the wave. Near the wall $\rho c > \eta k$. The density of the gas reduces with the radius. At $r < a_{min}$ this inequality fails. This gives equation for a_{min} in the form

$$c\rho_w \exp\left(\frac{M\omega^2 a^2}{2RT_0} \left(\frac{a_{min}^2}{a^2} - 1\right)\right) = k\eta. \quad (15)$$

In the region $0 < r < a_{min}$ the amplitude of the wave is close to zero due to strong viscosity. In the interval between a_{min} and a the wave velocity is described by eq. (4). The temperature

perturbation is also described by solution from [5]. Therefore it is naturally use the solution (4) for calculation of the volume damping but cutting the integral below a_{min} . Equation (4) does not take into account the interaction with the wall. Therefore the integration on r can be extended from $a - h$ to a because $h \ll a$. Then, integrating (2) in the interval from a_{min} to a we obtain the following equation for \dot{E}_{mec}

$$\begin{aligned} \dot{E}_{mec}^{vol} = & -\frac{\pi L}{2} \frac{\bar{p}_w^2}{\rho_w^2} \left(\left(\frac{\lambda}{c_p^2 T_0} + \frac{\eta}{c^2} \right) \left(2A\Gamma + 1 + \frac{1}{A} \left(\frac{k\eta}{c\rho_w} \right)^{-2\Gamma} \ln \frac{k\eta}{c\rho_w} + \right. \right. \\ & \left. \left. + k^2 \frac{a^2}{2A\Gamma} \left(\frac{\lambda}{c_p^2 T_0} + \frac{4}{3} \frac{\eta}{c^2} \right) \left(1 - \left(\frac{k\eta}{c\rho_w} \right)^{-2\Gamma} \right) \right), \end{aligned} \quad (16)$$

where $\Gamma = \frac{\gamma-1}{\gamma}$. Energy of the wave E has been calculated according to eq.(6). Then, volume damping coefficient equals

$$\begin{aligned} \gamma_d^{vol} = & \frac{|\dot{E}_{mec}^{vol}|}{2Ec} = \frac{A}{a^2} \frac{2-\gamma}{\gamma} \left(1 - \exp \left(-A \frac{(2-\gamma)}{\gamma} \right) \right)^{-1} \cdot \\ & \cdot \left(\left(\frac{\lambda}{c_p^2 T_0} + \frac{\eta}{c^2} \right) \left(2A\Gamma + 1 + \frac{1}{A} \left(\frac{k\eta}{c\rho_w} \right)^{-2\Gamma} \ln \frac{k\eta}{c\rho_w} + \right. \right. \\ & \left. \left. + k^2 \frac{a^2}{2A\Gamma} \left(\frac{\lambda}{c_p^2 T_0} + \frac{4}{3} \frac{\eta}{c^2} \right) \left(1 - \left(\frac{k\eta}{c\rho_w} \right)^{-2\Gamma} \right) \right). \end{aligned} \quad (17)$$

3. Conclusions

The analytical equation for the damping coefficient of the acoustic wave in the gas placed in to the strong centrifugal field is determined. The total damping consists of two part: the volume and surface damping. The volume damping is defined by the gradients of the velocity and temperature in the volume of the gas. The surface damping is defined by interaction of the wave with the wall of the rotor. Surface dumping strongly dominates the volume damping at the conventional parameters of the GC. Comparison with our numerical experiments [12] shows that our analytical equation agrees with numerical results in the limits of a few percents in the frequency range conventional for GC. The accuracy of estimation of the volume damping is essentially worse. But since the surface damping strongly dominates the volume damping, the total damping is estimated practically with the same accuracy in a few percents appropriate for the practical exploration.

Acknowledgments

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