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# Topological properties of Josephson current between two $s$ - and $p$ -wave superconducting nanowires with Majorana fermions

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**Abstract.** Josephson current between two one-dimensional nanowires with proximity induced either  $s$ -wave or  $p$ -wave pairing and separated by a narrow dielectric barrier is calculated in the presence of Rashba spin-orbit interaction, in-plane and normal Zeeman magnetic fields. We show that Andreev retro-tunneling is realized by means of two channels. The main contribution to the Josephson current in junction of  $s$ - or  $p$ -wave superconductors is shown to give Andreev scattering in a conventional particle-hole channel, when an electron-like quasiparticle reflects to a hole-like quasiparticle with opposite spin yielding a current which depends only on the order parameters' phase differences  $\varphi$  and oscillates with fractional  $4\pi$  period. Nevertheless Andreev bound state energy in second anomalous particle-hole channel, corresponding to the reflection of an incident electron-like quasiparticle to a hole-like quasiparticle with the same spin orientation, survives only in the presence of the in-plane magnetic field, and oscillates with  $4\pi$  period not only with  $\varphi$  but also with orientational angle of the in-plane magnetic field  $\theta$  resulting in a magneto-Josephson effect. In the presence of Rashba spin-orbit coupling (SOC) and normal-to-plane magnetic field  $h$ , a forbidden gap is shown to open in the dependence of Andreev bound state energies on the phases  $\varphi$  and  $\theta$  at several values of SOC strength and magnetic field, where Josephson current seems to vanish. The magnetic fields destroy also the particle-hole symmetry of the mid-gap states around Fermi level.

## 1. Introduction

Majorana zero modes, which are excitations at zero energy and are localized at interface of the topological/non-topological interface, are firstly predicted by Kitaev [1] to emerge in a spinless  $p$ -wave superconductor (SC). Recently it was suggested that a topological (spinless  $p$ -wave) superconductivity can be effectively realized either in a spin-polarized normal metal or in a semiconductor nanowire with strong spin-orbit coupling under Zeeman magnetic field proximity-coupled to a conventional spin-singlet ( $s$ -wave) bulk SC. One of the virtues of this model is that the proximitized nanowire can be driven into a topological phase by tuning the magnetic field or the chemical potential. The scientists believe that the topological SCs (TSCs) can be applied to quantum information processing, and a quantum information unit, qubit, can be formed and propagated by means of Majorana modes [2]. Recent investigations suggest several detection mechanisms of Majorana fermions (MFs) [3] such as an existence of a central



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peak in the tunneling current through a TSC (S) normal metal (N) junction and fractional period of Josephson current in S-N-S junctions.

In this work we study Josephson junction ( $JJ$ ) of two superconductors with  $s$ - or  $p$ -wave pairing of spinfull electrons separated by a  $\delta$ -function like insulator potential. Our paper is the first attempt where effects of Rashba SOC and Zeeman magnetic field on the Andreev bound state energy and Josephson current in the  $JJ$  produced by  $s$ - or  $p$ -wave superconductors are analytically and numerically studied. We show in this paper that the Andreev reflection is realized by means of two channels, and clarify the origin of these channels. Apart from the conventional Andreev reflection, when a quasi-particle reflects at the interface as a quasi-hole with opposite spin orientation, there appears a second anomalous particle-hole channel, corresponding to the Andreev reflection of an incident electron-like quasiparticle to a hole-like quasiparticle with the same spin orientation. This anomalous reflection channel is shown to survive only in the presence of the in-plane magnetic field, and contribution to the current in this channel oscillates with order parameter difference  $\varphi$  as well as with the in-plane magnetic field orientation angle  $\theta$  with  $4\pi$  fractional period which is referred to as *magneto-Josephson* current. Although the magneto-Josephson effect has been predicted for  $JJ$  of  $s$ -wave superconductors, we show that this effect takes place in the  $JJ$  of spinfull  $p$ -wave superconductors too [5]. We find the dependence of mid-gap energy expressions on Rashba SOC constant and external magnetic field analytically for several asymptotic cases as well as numerically for the general case. Zeeman magnetic field, which is normal to the wire's plane, destroys the quasi-particle/quasi-hole symmetry, and results in an asymmetric oscillation of the Josephson current. We show that Andreev bound state energy and Josephson current vanishes in some interval of the order parameter phase difference in the presence of Rashba SOC and Zeeman magnetic field, i.e. a forbidden gap is opened in the dependence of Josephson current on the phases at some definite values of SOC and magnetic fields.

## 2. Model and formulation of the problem

We consider a junction of two 1D nanowires of proximity induced  $s$ - or  $p_x$ -wave pairing symmetry superconductivity, in the presence of Rashba SOC and external Zeeman magnetic fields. Hamiltonian for such a system reads

$$\hat{H} = \hat{H}_{SC} + \hat{H}_R, \quad (1)$$

where

$$\begin{aligned} \hat{H}_{SC} = \int dx \sum_{\sigma, \sigma'} \left\{ \psi_{\sigma}^{\dagger}(x) \left[ [\xi_{\hat{k}} + U(x)]\sigma_0 + h\sigma_z + B\{\sigma_x \cos \theta_L + \sigma_y \sin \theta_L\}\theta(-x) \right. \right. \\ \left. \left. + [\sigma_x \cos \theta_R + \sigma_y \sin \theta_R]\theta(x) \right] \psi_{\sigma'}(x) + \psi_{\sigma}^{\dagger}(x) \left[ (\Delta_{\sigma, \sigma'})_L \theta(-x) + (\Delta_{\sigma, \sigma'})_R \theta(x) \right] \psi_{\sigma'}^{\dagger}(x) \right. \\ \left. + \text{h.c.} \right\}, \quad (2) \end{aligned}$$

where  $\xi_{\hat{k}} = \epsilon \left( \frac{\hbar}{i} \frac{\partial}{\partial x} \right) - \epsilon_F$  denotes the electron kinetic energy as measured from the Fermi level  $\epsilon_F$ ,  $\psi_{\sigma}(x)$  is the electron annihilation operator,  $h$  and  $\mathbf{B}$  are external Zeeman magnetic fields in  $z$  direction and in the  $x$ - $y$  plane respectively, and  $\sigma_0$ ,  $\sigma_{x,y,z}$  denote Pauli matrices. We choose  $\mathbf{B}$  in the left side of the junction to be aligned along the wire ( $\theta_L = 0$ ) while in the right side it is chosen to make an angle  $\theta$  with it ( $\theta_R = \theta$ ). The order parameter  $\Delta_{\sigma, \sigma'}$  in equation (2) for singlet and triplet pairings has the form  $\Delta_{\sigma, \sigma'} = \Delta_a g(k)(i\sigma_y)_{\sigma, \sigma'}$  and  $\Delta_{\sigma, \sigma'}(\mathbf{k}) = \Delta_a \mathbf{d}(\mathbf{k})(i\sigma\sigma_y)_{\sigma, \sigma'}$  correspondingly. The pairing potential  $\Delta_R$  in the right ( $a = R$ ) of the junction is chosen to have a phase difference  $\varphi$  compared to its left ( $a = L$ ) counterpart:  $\Delta_R = |\Delta| \exp(i\varphi)$  and  $\Delta_L = |\Delta|$ .

The potential  $U(x) = U_0\delta(x)$ , located at  $x = 0$ , represents the barrier potential between two superconductors. The Hamiltonian of Rashba SOI can be written as

$$\hat{H}_R = \sum_{\sigma, \sigma'} \int dx \psi_{\sigma}^{\dagger}(x) \alpha [v_x \sigma_z] \psi_{\sigma'}(x), \quad (3)$$

where  $\alpha$  is the strength of Rashba SOI. The order parameters  $\Delta_{a,\sigma}(x, k_x)$  for 1D  $s$ - and  $p_x$ -wave SCs can be simplified as

$$\Delta_{a,\sigma}(x, k_x) = \begin{cases} \sigma \Delta_a & \text{for } s\text{-wave SC,} \\ \Delta_a \frac{k_x}{k_F} & \text{for } p\text{-wave SC.} \end{cases} \quad (4)$$

### 3. Andreev bound states, Josephson and magneto-Josephson effects

In order to obtain a solution for the Andreev bound states for the junction described by equation (2) one follows the method used in [4]. Selection, e.g.  $\sigma = \downarrow$  ( $\sigma = \uparrow$ ) for a quasiparticle spin and  $\sigma' = \uparrow$  ( $\sigma' = \downarrow$ ) for a quasi-hole spin, provides the following expression for the determinants  $F_{\uparrow\downarrow}^s$  in  $s$ -wave  $JJ$

$$F_{\uparrow\downarrow}^s(k) = \left\{ [\alpha k M_+(k) + 2(v_F k - \alpha k)(E^2 + \alpha^2 k^2)]^2 - 4D|\Delta|^2(E^2 + \alpha^2 k^2)^2 \sin^2 \frac{\varphi}{2} \right\} (|\Delta|^2(E^2 + \alpha^2 k^2)^2)^{-1}, \quad (5)$$

and  $F_{\uparrow\downarrow}^p$  in  $p$ -wave  $JJ$ ,

$$F_{\uparrow\downarrow}^p = -\frac{4e^{-i\varphi}}{|\Delta|^2 M_-^2} \left\{ [(E+h)M_- + 2B^2 E]^2 - D \cos^2 \frac{\varphi}{2} \times \left[ [(E+h)M_- - 2B^2 E]^2 + [k(v_F - \alpha)M_- + 2B^2 \alpha k]^2 \right] \right\}, \quad (6)$$

where

$$M_{\pm} = (E \pm h)^2 + (v_F \mp \alpha)^2 k^2 + B^2 - |\Delta|^2. \quad (7)$$

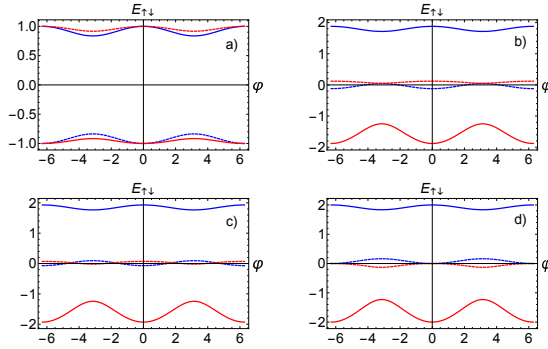
Now we choose  $\sigma = \sigma' = \uparrow$  ( $\sigma = \sigma' = \downarrow$ ) for the quasiparticle and quasi-hole spin orientations in order to get an evident expression for the determinant  $F_{\uparrow\uparrow}^{s,p}$  in the case of Andreev reflection in anomalous particle-hole reflection channel.  $F_{\uparrow\uparrow}^s$  in  $s$ -wave  $JJ$

$$F_{\uparrow\uparrow}^s(k) = \frac{16B^2}{|\Delta|^2 M_+^2(k)} \left\{ (E v_F k + \alpha k h)^2 - D|\Delta|^2 \times (E^2 + \alpha^2 k^2) \sin^2 \frac{\varphi - \phi}{2} \right\}. \quad (8)$$

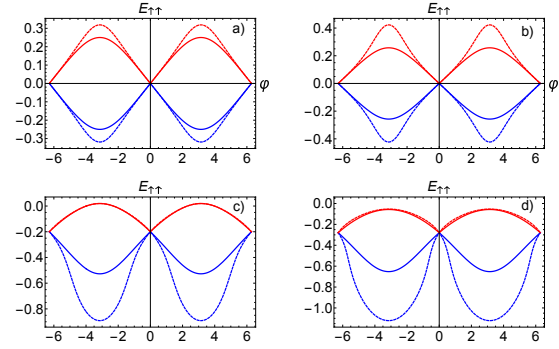
and also of  $p$ -wave  $JJ$ ,

$$F_{\uparrow\uparrow}^p = \frac{16B^2 e^{-i(\varphi-\theta)}}{|\Delta|^2 M_+^2} \left\{ (E k v_F + \alpha h k)^2 - |\Delta|^2 (E^2 + \alpha^2 k^2) \left[ 1 - D \cos^2 \frac{(\varphi - \theta)}{2} \right] \right\}. \quad (9)$$

Equation  $F_{\uparrow\uparrow}^{s,p} = 0$  provides the condition to find the Andreev bound-state energy in the anomalous particle-hole channel. The general feature of the Andreev bound-state energy in the anomalous particle-hole reflection channel  $E_{\uparrow\uparrow}^{s,p}$  with the same spin orientation is that it takes non-zero values only in the presence of in-plane magnetic field  $\mathbf{B}$ . Furthermore, it depends on the angle  $\theta$  between the junction and in-plane magnetic field. Oscillation of the Josephson current with  $\theta$  results in a fractional magneto-Josephson effect.



**Figure 1.** Andreev bound state energy  $E_{\uparrow\downarrow}^s$  in the particle-hole channel for  $\mathbf{B} = 0$ ,  $D_{\uparrow\downarrow} = 0.3$ ,  $\alpha = 0.2$  and different values of magnetic field (a)  $h = 0.0$ , (b)  $h = 0.88$ , (c)  $h = 0.93$ , (d)  $h = 1.0$ .



**Figure 2.** Andreev bound state energy  $E_{\uparrow\uparrow}^s$  in the anomalous particle-hole channel for  $D = 0.3$ ,  $\alpha = 0.4$  and (a)  $\mathbf{B} = 0.2$ ,  $h = 0.0$ , (b)  $\mathbf{B} = 0.5$ ,  $h = 0.0$ , (c)  $\mathbf{B} = 0.2$ ,  $h = 0.5$ , (d)  $\mathbf{B} = 0.2$ ,  $h = 0.7$ .

Andreev bound state energy can be numerically calculated according to equations (5)–(9) for arbitrary values of  $\alpha$ ,  $\mathbf{B}$  and  $h$ . Figure 1 plots Andreev bound state energy vs  $\varphi$  for  $s$ -wave  $JJ$  according to the condition  $F_{\uparrow\downarrow}^s = 0$  with equation (5) for  $\mathbf{B} = 0$  and non-zero values of Rashba SOC constant  $\alpha \neq 0$  at different values of normal magnetic field  $h$ . It is clearly seen from the figure 1 that the energies of one particle and hole branches decrease with increasing  $h$ , and touch each other at  $h = h_c = 0.93$ , where Majorana particle is localized. Further increase in  $h$  results in opening a forbidden gap in the dependence of  $E_{\uparrow\downarrow}^s$  on  $\varphi$ .

The mid-gap energy  $E_{\uparrow\uparrow}^s$  for  $s$ -wave  $JJ$  in the anomalous particle-hole channel is drawn in figure 2 for non-zero values of all parameters  $\alpha \neq 0$ ,  $\mathbf{B} \neq 0$  and  $h \neq 0$ . The magnetic fields destroy the symmetry between the particle and hole energy branches, the normal magnetic field  $h$  shifts asymmetrically both energy branches up or down.

#### 4. Conclusion

In this paper we have studied Josephson junction in  $s$ - or  $p$ -wave  $JJ$  in the presence of Rashba SOC and Zeeman magnetic fields  $\mathbf{B}$  and  $h$ . Andreev retro-reflection is shown to be realized in two channel. We showed that apart from the conventional particle-hole reflection, a new anomalous reflection channel appears, when a quasiparticle reflects to a quasi-hole state with the same spin orientation, which are responsible to the magneto-Josephson effect. Note that Josephson current can be calculated from the expression

$$J = \frac{2e}{\hbar} \sum_{a,\sigma} \frac{\partial E_{a,\sigma}}{\partial \varphi}, \quad (10)$$

which can provide an evident dependence of Josephson current on Rashba SOC constant, and Zeeman magnetic fields  $\mathbf{B}$  and  $h$ .

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