**PAPER • OPEN ACCESS**

# Analytical reconstruction of *pp* elastic scattering amplitudes from the complete sets of experiments at the SPASCHARM facility at U70

To cite this article: A A Bogdanov *et al* 2020 *J. Phys.: Conf. Ser.* **1690** 012084

# Analytical reconstruction of *pp* elastic scattering amplitudes from the complete sets of experiments at the SPASCHARM facility at U70

**A A Bogdanov[1], V A Chetvertkova[4], A V Kozelov[2], V P Ladygin[3], V V Mochalov[1,2], M B Nurusheva [1], V A Okorokov[1], A O Oreshkov[1,2], V L Rykov[1], P A Semenov[1,2] and A N Vasiliev[1,2]**

[1] National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Moscow, 115409, Russia
[2] NRC «Kurchatov Institute» - IHEP, Protvino, Moscow region, 142281, Russia
[3] Joint Institute for Nuclear Research, Dubna, Moscow region, 141980,Russia
[4] GSI, Plankstrasse 1, 64291, Darmstadt, Germany

E-mail: mbnurusheva@mail.ru

**Abstract.** The direct reconstruction of the pp elastic scattering amplitudes at the energy of 16 GeV is discussed. At the SPASCHARM experiment, the 19 different spin dependent observables will be measured in *pp* elastic scattering. The suggested selection of observables allows a complete analytical reconstruction of elastic scattering amplitudes as a solution of the system of bilinear equations. A set of physical observables, which are necessary for model independent reconstruction of all five complex elastic pp scattering amplitudes, is defined.

## 1. Introduction

A direct reconstruction of the scattering amplitudes for a given reaction is the only completely model independent way of extracting the maximal possible information from a set of experiments. In particular, for elastic proton-proton scattering, the imposing of Lorentz invariance, parity, time reversal invariance and the Pauli principle limits the set to only five complex scattering amplitudes. Each of them is a function of energy and scattering angle.

Before 1975, the direct reconstruction from available experimental data was done only for a few energies at 90° CM [1], and the amplitudes were found with relatively large errors. Later on, the statistical approach for reconstruction of scattering amplitudes has been developed [2]. Statistical approach does not imply a direct solution of the equations, but the experimental results are fitted by varying scattering amplitudes.

The first direct reconstruction of the pp scattering matrix over a quite large angular range was reported in [2, 3]. At PSI, for the energies below 0.6 GeV, an unambigous direct reconstruction of scattering matrices has been successfully done from the complete sets of 17 measured observables, as well as the time-reversal invariance has been tested [3, 4]. But for the energies above 6 GeV, the complete scattering matrices for elastic scattering had never been reconstructed. A comprehensive overview of the respective methods, along with experimental results, can be found in [5].

Spin physics program of the "Kurchatov institute" - IHEP [6] will provide one more opportunity for studying an elastic nucleon scattering. The new beamline will provide an unique opportunity to have both polarized proton and antiproton beams with the intensity of up to $10^7$ and $10^6$ particles per 10 sec cycle, respectively [7]. An extensive physics program of the fixed target experiment SPASCHARM (SPin ASymmetry in CHARMonia) is described in detail in the Conceptual Design Report [8]. The experiment program covers the study of spin effects in dozens of inclusive and exclusive hadronic reactions.

The beam polarimetry is an inevitable part of the SPASCHARM experiment [9]. It relies on the known spin asymmetries in some physics processes, and elastic scattering is the very important one among such processes of interest [10]. In turn, and availability of polarized beams, interacting with a number of different polarized and unpolarized targets, provides the tools for a comprehensive physics study of elastic scattering itself [11]. Ultimately, the full set of elastic spin-dependent amplitudes [12] can be measured in $pp$ and $p\bar{p}$ collisions at 16 GeV.

The choice of the measured observables essentially depends on the concrete experimental equipment and on the performance of the corresponding accelerator and its energy. The SPASCHARM configuration, when polarizations of protons beam [7] and target [13] can be oriented along any of three directions, allow us to measure 19 non-vanishing observables. The schematic view of the SPASCHARM absolute polarimeter is presented in figure 1 [14].

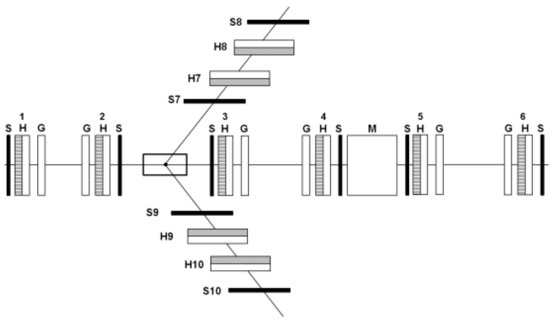



**Figure 1.** Schematic view of the SPASCHARM absolute polarimeter (upper view).

Throughout the paper, we use the Nucleon-Nucleon (NN) formalism and the four-index notation for observables given in [15]. We use the scattering matrix in the form

$$M(\mathbf{k}',\mathbf{k}) = 1/2\{(a+b)+(a-b)(\sigma_1,\mathbf{n})(\sigma_2,\mathbf{n})+(c+d))(\sigma_1,\mathbf{m})(\sigma_2,\mathbf{m})+ \tag{1.1}$$

$$+(c-d)(\sigma_1,\mathbf{l})(\sigma_2,\mathbf{l})+e(\sigma_1+\sigma_2,\mathbf{n})\},$$

$$\mathbf{n}=[\mathbf{k}'\times\mathbf{k}]/|[\mathbf{k}'\times\mathbf{k}]|,\ \mathbf{l}=(\mathbf{k}'+\mathbf{k})/|\mathbf{k}'+\mathbf{k}|,\ \mathbf{m}=(\mathbf{k}'-\mathbf{k})/|\mathbf{k}'-\mathbf{k}|, \tag{1.2}$$

where $a$, $b$, $c$, $d$ and $e$ are the scattering amplitudes, $\sigma_1$ and $\sigma_2$ are the Pauli 2 x 2 matrices, $\boldsymbol{k}$ and $\boldsymbol{k'}$ are the unit vectors in the directions of the incident and scattered particles, respectively. The subscripts of any observable $X_{srbt}$ refer to the polarization states of the scattered, recoil, beam, and target particles, respectively. The polarizations of the incident and target particles are oriented along unit vectors $\boldsymbol{n}$, $\boldsymbol{s}$, and $\boldsymbol{k}$ for the beam and target laboratory frame:

$$\mathbf{k},\quad \mathbf{n}=[\mathbf{k}\times\mathbf{k}'],\quad \mathbf{s}=[\mathbf{n}\times\mathbf{k}]; \tag{1.3}$$

and $\boldsymbol{n}$, $\boldsymbol{s}''$, and $\boldsymbol{k}''$ for the recoil particle frame:

$$\mathbf{k}'',\quad \mathbf{n},\quad \mathbf{s}''=[\mathbf{n}\times\mathbf{k}'']. \tag{1.4}$$

## 2. Relations between contributing observables and amplitudes

In the following equations, we provide the relations between scattering amplitudes *a*, *b*, *c*, *d*, *e* and observables at any angle. We denote by $\theta$ the CM scattering angle and by $\theta_2$ - the laboratory angle of the recoil particle.

$$\sigma = \frac{d\sigma}{d\Omega} = \frac{1}{2}|a|^2 + \frac{1}{2}|b|^2 + \frac{1}{2}|c|^2 + \frac{1}{2}|d|^2 + \frac{1}{2}|e|^2 \tag{2.1}$$

$$A_3 = \sigma A_{oonn} = \frac{1}{2}|a|^2 - \frac{1}{2}|b|^2 - \frac{1}{2}|c|^2 + \frac{1}{2}|d|^2 + \frac{1}{2}|e|^2 \tag{2.2}$$

$$K_3 = \sigma K_{onno} = \frac{1}{2}|a|^2 - \frac{1}{2}|b|^2 + \frac{1}{2}|c|^2 - \frac{1}{2}|d|^2 + \frac{1}{2}|e|^2 \tag{2.3}$$

$$D_3 = \sigma D_{onon} = \frac{1}{2}|a|^2 + \frac{1}{2}|b|^2 - \frac{1}{2}|c|^2 - \frac{1}{2}|d|^2 + \frac{1}{2}|e|^2 \tag{2.4}$$

$$P = \sigma A_{oono} = \sigma A_{ooon} = \mathrm{Re}(a^* e) \tag{2.5}$$

$$N_2 = \sigma N_{onkk} = -\mathrm{Re}(d^* e)\cos(\theta) - \mathrm{Im}(a^* d)\sin(\theta) \tag{2.6}$$

$$A_1 = \sigma A_{oosk} = -\mathrm{Im}(d^* e)\cos(\theta) - \mathrm{Re}(a^* d)\sin(\theta) \tag{2.7}$$

$$N_1 = \sigma N_{onsk} = -\mathrm{Re}(d^* e)\sin(\theta) + \mathrm{Im}(a^* d)\cos(\theta) + \mathrm{Im}(b^* c) \tag{2.8}$$

$$N_7 = \sigma N_{onks} = -\mathrm{Re}(d^* e)\sin(\theta) + \mathrm{Im}(a^* d)\cos(\theta) - \mathrm{Im}(b^* c) \tag{2.9}$$

$$A_2 = \sigma A_{ookk} = \mathrm{Im}(d^* e)\sin(\theta) - \mathrm{Re}(a^* d)\cos(\theta) + \mathrm{Re}(b^* c) \tag{2.10}$$

$$A_4 = \sigma A_{ooss} = -\mathrm{Im}(d^* e)\sin(\theta) + \mathrm{Re}(a^* d)\cos(\theta) + \mathrm{Re}(b^* c) \tag{2.11}$$

$$K_2 = \sigma K_{os"so} = -\mathrm{Re}(a^* c)\cos(\theta+\theta_2) + \mathrm{Im}(c^* e)\sin(\theta+\theta_2) - \mathrm{Re}(b^* d)\cos(\theta_2) \tag{2.12}$$

$$N_4 = \sigma N_{os"kn} = -\mathrm{Im}(a^* c)\cos(\theta+\theta_2) + \mathrm{Re}(c^* e)\sin(\theta+\theta_2) + \mathrm{Im}(b^* d)\cos(\theta_2) \tag{2.13}$$

$$K_1 = \sigma K_{os"ko} = \mathrm{Re}(a^* c)\sin(\theta+\theta_2) + \mathrm{Im}(c^* e)\cos(\theta+\theta_2) - \mathrm{Re}(b^* d)\sin(\theta_2) \tag{2.14}$$

$$N_5 = \sigma N_{os"sn} = -\mathrm{Im}(a^* c)\sin(\theta+\theta_2) - \mathrm{Re}(c^* e)\cos(\theta+\theta_2) - \mathrm{Im}(b^* d)\sin(\theta_2) \tag{2.15}$$

$$D_1 = \sigma D_{os"ok} = \mathrm{Re}(a^* b)\sin(\theta+\theta_2) - \mathrm{Re}(c^* d)\sin(\theta_2) + \mathrm{Im}(b^* e)\cos(\theta+\theta_2) \tag{2.16}$$

$$D_2 = \sigma D_{os"os} = -\mathrm{Re}(a^* b)\cos(\theta+\theta_2) - \mathrm{Re}(c^* d)\cos(\theta_2) + \mathrm{Im}(b^* e)\sin(\theta+\theta_2) \tag{2.17}$$

$$N_3 = \sigma N_{os"nk} = -\mathrm{Im}(a^* b)\cos(\theta+\theta_2) + \mathrm{Im}(c^* d)\cos(\theta_2) + \mathrm{Re}(b^* e)\sin(\theta+\theta_2) \tag{2.18}$$

$$N_6 = \sigma N_{os"ns} = -\mathrm{Im}(a^* b)\sin(\theta+\theta_2) - \mathrm{Im}(c^* d)\sin(\theta_2) - \mathrm{Re}(b^* e)\cos(\theta+\theta_2) \tag{2.19}$$

Both SPASCHARM beam and the target will be polarized along ***n***, ***s*** or ***k*** directions, and the components ***n*** and ***s″*** of the recoil particles will be analyzed. It will allow us to measure the following non-vanishing observables:

$A_{oono}$, $A_{oono}$ - beam and target analyzing power, respectively;

$A_{oonn}$, $A_{ooss}$, $A_{ookk}$, $A_{oosk}$ - four spin correlation parameters.

The polarization of the recoil proton is the same as analyzing power: $P_{onoo} = A_{oono} = A_{ooon} = P$.

In addition, the following rescattering observables will be measure:
$K_{onno}$ , $K_{os"so}$ , $K_{os"ko}$ - three polarization transfer coefficients from the beam to the recoil particle;
$D_{onon}$ , $D_{os"ok}$ $D_{os"os}$ - three depolarization coefficients for the target;
$N_{onkk}$ , $N_{onsk}$ , $N_{onks}$ , $N_{os"ns}$ , $N_{os"nk}$ , $N_{os"kn}$ , $N_{os"sn}$ - seven polarizations of the recoil particle.
Thus, the 19 different observables in total can be measured at U70.

## 3. Direct reconstruction from 13 experiments

Assuming that all 19 experimental observables from equations (2.1) to (2.19) were measured with a sufficient precision, we can choose any complete set from them. First, we define 11 most convenient observables:

$$\{\sigma, P, A_3, D_3, K_3, A_1, A_2, A_4, N_1, N_2, N_7\} \tag{3.1}$$

We choose the amplitude $e$ to be real and positive and introduce the notations for real and imaginary parts of each complex amplitude: Re $e = e$, Im $e = 0$ , $a = a_1 + ia_2$ , $b = b_1 + ib_2$, $c = c_1 + ic_2$, $d = d_1 + id_2$ .

The solution for $a_1$ from (2.5) is:

$$a_1 = \frac{P}{e} \tag{3.2}$$

Using equations (2.1)–(2.4), we obtain:

$$\sigma + A_3 + D_3 + K_3 = 2|a|^2 + 2|e|^2 \tag{3.3}$$

From two last equations, we find $a_2$:

$$a_2 = [(e^2(\sigma + A_3 + D_3 + K_3) - 2e^4 - 2P^2) / 2e^2]^{1/2} \tag{3.4}$$

Taking the sum (2.10) and (2.11), we obtain:

$$\mathrm{Re}(b^* c) = \frac{A_4 + A_2}{2} \tag{3.5}$$

The subtraction of (2.9) from (2.8) results in:

$$\mathrm{Im}(b^* c) = \frac{N_1 - N_7}{2} \tag{3.6}$$

We express $d_1$ in terms of *e*, by composing (2.8), (2.9) and (2.6), multiplying by $\sin(\theta)$ and $\cos(\theta)$ :

$$d_1 = \frac{-(N_1 + N_7)\sin(\theta) - 2N_2\cos(\theta)}{2e} \tag{3.7}$$

We express $d_2$ in terms of *e*, composing (2.10), (2.11) and (2.7) and multiplying by $\sin(\theta)$ and $\cos(\theta)$ :

$$d_2 = \frac{(A_4 - A_2)\sin(\theta) + 2A_1\cos(\theta)}{2e} \tag{3.8}$$

Finally, we find amplitude $e$ from (3.7) and (3.8) by substituting them into the expression $\sigma + A_3 - D_3 - K_3 = 2|d|^2$ :

$$e = [(\sigma + A_3 - D_3 - K_3)((4(A_1^2 + N_2^2) - (A_4 - A_2)^2 - (N_1 + N_7)^2)\cos(2\theta) + \\ +(4A_1(A_4 - A_2) + 4N_2(N_1 + N_7))\sin(2\theta) + 4(A_1^2 + N_2^2) + (A_4 - A_2)^2 + \\ +(N_1 + N_7)^2)]^{1/2} / 2(\sigma + A_3 - D_3 - K_3) \tag{3.9}$$

It is necessary to add to the set (3.1) two more observables, $D_1$ and $K_1$, to determine amplitudes $b_1$, $b_2$, $c_1$ and $c_2$, by solving linear equations, using (3.5) and (3.6):

$$\{\sigma, P, A_3, D_3, D_1, K_3, K_1, A_1, A_2, A_4, N_1, N_2, N_7\} \tag{3.10}$$

We introduce two more notations $L_1$ and $L_2$ and express $c_1$ and $c_2$ in terms of $b_1$, $b_2$, $L_1$ and $L_2$:

$$L_1 = b_1c_1 + b_2c_2 = \frac{A_2 + A_4}{2},\ L_2 = b_1c_2 - b_2c_1 = \frac{N_1 - N_7}{2} \tag{3.11}$$

$$c_1 = \frac{L_1b_1 - L_2b_2}{|b|^2},\ c_2 = \frac{L_1b_2 + L_2b_1}{|b|^2} \tag{3.12}$$

We obtain a system of two linear equations for $b_1$, and $b_2$: using equations (2.14) и (2.16) and taking into account (3.11) and (3.12)

$$K_1|b|^2 = \{(a_1L_1 + a_2L_2)\sin(\theta + \theta_2) - |b|^2 d_1\sin(\theta_2) - eL_2\cos(\theta + \theta_2)\}b_1 + \\ +\{(a_2L_1 - a_1L_2)\sin(\theta + \theta_2) - |b|^2 d_2\sin(\theta_2) - eL_1\cos(\theta + \theta_2)\}b_2 \tag{3.13}$$

$$D_1|b|^2 = \{|b|^2 a_1\sin(\theta + \theta_2) - (d_1L_1 + d_2L_2)\sin(\theta_2)\}b_1 + \\ +\{|b|^2(a_2\sin(\theta + \theta_2) - e\cos(\theta + \theta_2)) + (d_1L_2 + d_2L_1)\sin(\theta_2)\}b_2 \tag{3.14}$$

We denote coefficients for $b_1$, and $b_2$ as $Q$, $R$, $S$ and $T$ and solve the system of linear equations:

$$Q = (a_1L_1 + a_2L_2)\sin(\theta + \theta_2) - |b|^2 d_1\sin(\theta_2) - eL_2\cos(\theta + \theta_2) \tag{3.15}$$

$$R = (a_2L_1 - a_1L_2)\sin(\theta + \theta_2) - |b|^2 d_2\sin(\theta_2) - eL_1\cos(\theta + \theta_2) \tag{3.16}$$

$$S = |b|^2 a_1\sin(\theta + \theta_2) - (d_1L_1 + d_2L_2)\sin(\theta_2) \tag{3.17}$$

$$T = |b|^2(a_2\sin(\theta + \theta_2) - e\cos(\theta + \theta_2)) + (d_1L_2 + d_2L_1)\sin(\theta_2) \tag{3.18}$$

$$\begin{cases} Qb_1 + Rb_2 = K_1|b|^2 \\ Sb_1 + Tb_2 = D_1|b|^2 \end{cases} \tag{3.19}$$

$$b_1 = \frac{|b|^2(D_1R - K_1T)}{SR - TQ},\ b_2 = \frac{|b|^2(D_1Q - K_1S)}{TQ - SR} \tag{3.20}$$

Let us express the coefficients in terms of the observables and the amplitude $e$, taking into account, that $a_1$, $a_2$, $d_1$, $d_2$, $e$ are known, and $|b|^2$ is defined by the expression $\sigma - A_3 + D_3 - K_3 = 2|b|^2$:

$$Q = \frac{P(A_2 + A_4) + (N_1 - N_7)[(e^2(\sigma + A_3 + D_3 + K_3) - 2e^4 - 2P^2)/2]^{1/2}}{2e}\sin(\theta + \theta_2) + \tag{3.21}$$
$$+\left(\sigma - A_3 + D_3 - K_3\right)\frac{\left(N_1 + N_7\right)\sin(\theta) + 2N_2\cos(\theta)}{4e}\sin(\theta_2) + \frac{e(N_7 - N_1)}{2}\cos(\theta + \theta_2)$$

$$R = \frac{(A_2 + A_4)[(e^2(\sigma + A_3 + D_3 + K_3) - 2e^4 - 2P^2)/2]^{1/2} - P(N_1 - N_7)}{2e}\sin(\theta + \theta_2) - \tag{3.22}$$
$$-\left(\sigma - A_3 + D_3 - K_3\right)\frac{\left(A_4 - A_2\right)\sin(\theta) + 2A_1\cos(\theta)}{4e}\sin(\theta_2) - \frac{e(A_2 + A_4)}{2}\cos(\theta + \theta_2)$$

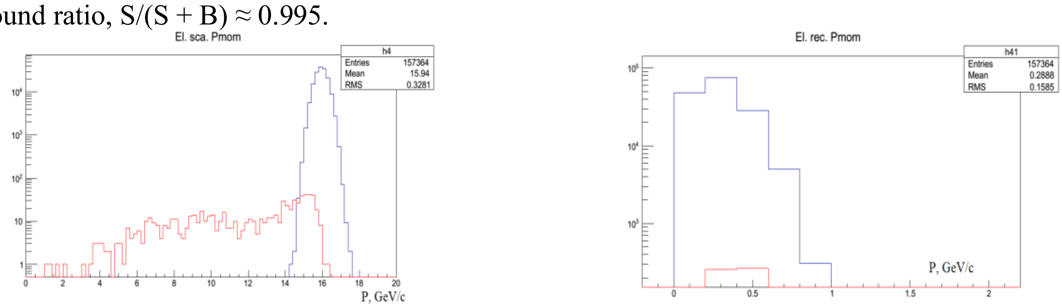
$$S = \{4\left(\sigma - A_3 + D_3 - K_3\right)P\sin(\theta + \theta_2) - [\left((N_1 + N_7)\sin(\theta) + 2N_2\cos(\theta)\right)(A_2 + A_4) \tag{3.23}$$
$$+\left((A_4 - A_2)\sin(\theta) + 2A_1\cos(\theta)\right)(N_1 - N_7)]\sin(\theta_2)\}/4e$$

$$T = \left([(e^2(\sigma + A_3 + D_3 + K_3) - 2e^4 - 2P^2)/2e^2]^{1/2}\sin(\theta + \theta_2) - e\cos(\theta + \theta_2)\right)\cdot \tag{3.24}$$
$$\cdot\left(\sigma - A_3 + D_3 - K_3\right) + \{[\left((N_7^2 - N_1^2) + (A_4^2 - A_2^2)\right)\sin(\theta)$$
$$+2\left(N_2(N_7 - N_1) + A_1(A_2 + A_4)\right)\cos(\theta)]\sin(\theta_2)\}/4e$$

We find $b_1$ and $b_2$, by substituting the coefficients $Q$, $R$, $S$ and $T$ in equation (3.20). Now, the equations for $c_1$ and $c_2$ in terms of the amplitude $e$ and the observables are expressed from (3.12).
Finally, we obtain all nine amplitudes from 13 observables without any ambiguities.

## 4. Possibility to study elastic scattering at SPASCHARM experiment

The possibility to study elastic scattering at the SPASCHARM experiment has been demonstrated earlier [16] with the fast Monte-Carlo. Figure 2 shows, that, using of the special selection criteria, we can significantly suppress background from diffraction events with a slight suppression of the signal. The estimated signal to background ratio, $S/(S + B) \approx 0.995$.




(a) (b)

**Figure 2.** The momentum distributions of the scattered (a) and recoil (b) particles. Blue line is for elastic processes, red line is for the diffraction [16].

## 5. Discussion and conclusion

Design of the SPASCHARM experiment setup for elastic scattering study allows us to measure 19 non-vanishing observables.

This made it possible to choose a set of observables convenient for reconstructing amplitudes in a laboratory system and greatly simplify calculations. We have demonstrated here that the set of 13 observables (3.10) is complete at any given energy and angle, and allows us to reconstruct all amplitudes of elastic scattering directly.

In the present article, we left out of scope the measurement uncertainties and reconstructed the amplitudes $a$, $b$, $c$, $d$ and $e$ analytically as if the observables were exactly known. The fast MC study has demonstrated the possibility to effectively suppress background and select a quite pure elastic process. The next step would be a full Monte-Carlo simulations with real description of the SPASCHARM setup in order to estimate the achievable statistical and systematic errors

**Acknowledgments**

The work was supported in part by RFBR, project number 18-02-00006 and the Ministry of Science and Higher Education of the Russian Federation, Project "Fundamental properties of elementary particles and cosmology" No 0723-2020-0041.

## References

[1] Lemon P *et al* 1968 *Phys. Rev.* **169** 1026
[2] Lechanoine-Leluc C and Lehar F 1993 *Rev. Mod. Phys.* **65** 47
[3] Aprile E *et al* 1981 *Phys. Rev. Lett.* **46** 1047
[4] McNaughton M W *et al* 1990 *Phys. Rev.* C **41** 2809
[5] Lehar F and Strokovsky E. 2010 *Phenomenology and Analysis of Data on Nucleon Scattering* (Moscow:Moscow State University Press) p 210
[6] Mochalov V 2013 *Physics of Particles and Nuclei* **44** 930
[7] Abramov V V *et a.l* 2018 *Nucl.Instrum.Meth.* **A901** 62
[8] Abramov V V *et al.* 2019 *IHEP Preprint* 2019-12 (in russian)
[9] Bogdanov A A *et al.* 2017 *J.Phys.Conf.Ser.* **798** 012179
[10] Bogdanov A A *et al.* 2016 *J.Phys.Conf.Ser.* **678** 012034
[11] Abramov V V *et al.* 2017 *J.Phys.Conf.Ser.* **938** 012006
[12] Bogdanov A A *et al* 2019 *J.Phys.Conf.Ser.* **1390** 012032
[13] Biroth M *et al.* 2015 "Design of the Mainz Active Polarized Proton Target" *PoS (PSTP2015)* 005
[14] Semenov P *et al.* 2016 *Int.J.Mod.Phys.Conf.Ser.* **40** *vol 1* 1660086
[15] Bystricky J, Lehar F and Winternitz P 1978 *Journal de Physique* **39** 1
[16] Bogdanov A A *et al.* 2020 *J.Phys.Conf.Ser.* **1435** 012044